Eigenvalues of Euclidean Distance Matrices and rs-majorization on $\mathbb{R}^2$

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Abstract

Let D_1 and D_2 be two Euclidean distance matrices (EDMs) with corresponding positive semidefinite matrices B_1 and B_2 respectively. Suppose that $\lambda(A) = ((\lambda(A))_i)_{i=1}^n$ is the vector of eigenvalues of a matrix A such that $(\lambda(A))_1 \geq \ldots \geq (\lambda(A))_n$. In this paper, the relation between the eigenvalues of EDMs and those of the corresponding positive semidefinite matrices respect to $\prec_{rs}$, on $\mathbb{R}^2$ will be investigated.

Keywords: Euclidean distance matrices, Rs-majorization.
Mathematics Subject Classification [2010]: 34B15, 76A10

1 Introduction

An $n \times n$ nonnegative and symmetric matrix $D = (d_{ij}^2)$ with zero diagonal elements is called a predistance matrix. A predistance matrix D is called Euclidean or a Euclidean distance matrix (EDM) if there exist a positive integer r and a set of n points $\{p_1, \ldots, p_n\}$ such that $p_1, \ldots, p_n \in \mathbb{R}^r$ and $d_{ij}^2 = \|p_i - p_j\|^2$ $(i, j = 1, \ldots, n)$, where $\|.\|$ denotes the usual Euclidean norm. The smallest value of r that satisfies the above condition is called the embedding dimension. As is well known, a predistance matrix D is Euclidean if and only if the matrix $B = \frac{-1}{2}PDP$ with $P = I_n - \dfrac{1}{n}ee^t$, where I_n is the $n \times n$ identity matrix, and e is the vector of all ones, is positive semidefinite matrix. Let Λ_n be the set of $n \times n$ EDMs, and $\Omega_n(e)$ be the set of $n \times n$ positive semidefinite matrices B such that $Be = 0$. Then the linear mapping $\tau : \Lambda_n \to \Omega_n(e)$ defined by $\tau(D) = \dfrac{-1}{2}PDP$ is invertible, and its inverse mapping, say $\kappa : \Omega_n(e) \to \Lambda_n$ is given by $\kappa(B) = be^t + eb^t - 2B$ with $b = diag(B)$, where $diag(B)$ is the vector consisting of the diagonal elements of B. For general refrence on this topic see, e.g. [1].

Majorization is one of the vital topics in mathematics and statistics. It plays a basic role in matrix theory. One can see some type of majorization in [2]-[13]. In this paper, the relation between the eigenvalues of EDMs and those of the corresponding positive

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