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SOME INEQUALITIES FOR THE NUMERICAL RADIUS OF OPERATORS

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Abstract

In this talk, we provide a generalization of a numerical radius inequality including product of two operators on a Hilbert space which is sharper than original inequality in a particular position. An application of this inequality to prove a numerical radius inequality that involves the generalized Aluthge transform is also given. In addition, our results generalize some known inequalities. For any $A, B, X \in \mathcal{B}(H)$ such that $A, B \geq 0$, we prepare new estimation for the numerical radius of two terms $A^\alpha X B^\alpha$, $A^\alpha X B^{1-\alpha}$ $(0 \leq \alpha \leq 1)$ and Heinz means. Other related inequalities are also discussed.

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1 Introduction

Recall that an operator $A \in \mathcal{B}(H)$ is called positive, denote by $A \geq 0$, if $\langle Ax, x\rangle \geq 0$ for all $x \in$ H. The numerical radius of $A \in \mathcal{B}(H)$ is defined by

$$w(A) = \sup\{|\lambda| : \lambda \in W(A)\},$$

where $W(A)$ is the numerical range of A defined by $W(A) = \{\langle Ax, x\rangle : x \in \mathrm{H}, \|x\| = 1\}$. For a comprehensive account of theory of the numerical range and numerical radius we refer the reader to [2].

It is well known that $w(\cdot)$ defines a norm on $\mathcal{B}(H)$ such that for all $A \in \mathcal{B}(H)$,

$$\frac{1}{2}\|A\| \leq w(A) \leq \|A\|. \tag{1}$$

On the second inequality in (1), Kittaneh [3] has shown that if $A \in \mathcal{B}(H)$, then

$$w(A) \leq \frac{1}{2}(\|A\| + \|A^2\|^{\frac{1}{2}}). \tag{2}$$

Obviously, inequality (2) is sharper than the second inequality of (1).
Inequalities (1) are sharp. If $A^2 = 0$, then $w(A) = \frac{1}{2}\|A\|$, while if A is normal, then $w(A) = \|A\|$. For $A \in \mathcal{B}(H)$, let $A = U|A|$ be the polar decomposition of A, the Aluthge

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