Generalized weighted composition operators between Zygmund spaces and Bloch spaces

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Abstract

For the analytic selfmap φ and analytic function u on the open unit ball of the complex plane, we investigate generalized weighted composition operators

$$\left(D^k_{\varphi,u}f\right)(z) = u(z)f^{(k)}(\varphi(z)),$$

between weighted Zygmund spaces and weighted Bloch spaces.

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1 Introduction

Let $\mathbb{D}$ be the open unit ball in $\mathbb{C}$ and u and φ be analytic functions on $\mathbb{D}$ such that $\varphi(\mathbb{D}) \subseteq \mathbb{D}$. For a nonnegative integer k, the *generalized weighted composition operator* $D^k_{\varphi,u}$ on $H(\mathbb{D})$, the space of all analytic functions on $\mathbb{D}$, is defined by

$$\left(D^k_{\varphi,u}f\right)(z) = u(z)f^{(k)}(\varphi(z)), \quad z \in \mathbb{D}.$$

Generalized weighted composition operators are generalization of well-known *weighted composition operators* uC_φ defined by

$$\left(uC_\varphi f\right)(z) = u(z)f(\varphi(z)), \quad z \in \mathbb{D},$$

and also generalization of some other known operators. In this paper, we consider generalized weighted composition operators between *weighted Zygmund spaces* and *weighted Bloch spaces* defined as follows.

By a *weight function* we mean a continuous, strictly positive and bounded function $\nu : \mathbb{D} \to \mathbb{R}_+$. The weight ν is called *radial* if $\nu(z) = \nu(|z|)$ for all $z \in \mathbb{D}$. For a weight ν, the *weighted Banach space of analytic functions on* $\mathbb{D}$ is defined as

$$H^\infty_\nu = \left\{ f \in H(\mathbb{D}) \ : \ \|f\|_\nu = \sup_{z\in\mathbb{D}} \nu(z)|f(z)| < \infty \right\}.$$

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