The infinite product representation of solutions of indefinite Sturm-Liouville problems with three turning points.

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Abstract

We study the infinite product representation of solutions of second order differential equation of Sturm-Liouville type on a finite interval having three turning points under the assumption that the turning points are types IV, II, III , respectively. Such representations are useful in the associated studies of inverse spectral problems for such equations.

Keywords: Turning point; Sturm-Liouville, Nondefinite problem; Infinite products, Hadamard Factorization Theorem; Spectral theory
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1 Introduction

The main purpose of the paper is to consider the infinite product representation of solutions of second order differential equation of Sturm-Liouville type on a finite interval of the form

$$y'' + (\lambda\phi^2(x) - q(x))y = 0, \qquad 0 \leq x \leq 1, \tag{1}$$

The functions $\phi^2(x)$ and $q(x)$ are referred to as the coefficients of the problem, the function $\phi^2(x)$ as the weight; they are real valued on the interval $(0, 1)$. The zeros of $\phi^2(x)$ (assumed to be a discrete set) are called the *turning points* or *transition points* (TP) of ((2)). The parameter λ is real.
The nature of the solutions of such Sturm-Liouville equation in the neighborhood of the turning points have been the object of humerous investigations. Readers interested in a historical survey on linear turning point theory are referred to the survey article of MCHUGH [13] .The results of Doronidcyn [2], McKelvey [7], Langer [5], Dyachenko [3], and Tumanov [11] bring important innovations to the asymptotic approximation of solutions of Sturm-Liouville equations with two turning points.

The representation of solutions of Sturm-Liouville equations by means of an infinite product is a direct consequence of the fact that any solution $y(x, \lambda)$ defined by a fixed set of initial conditions (as we have seen above) is necessarily an entire function of λ for each fixed $x \in [-1, 1]$, whose order does not exceed $1/2$ (see [1]). It follows from the

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