



The asymptotic average shadowing property for IFS

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Abstract

In this present paper we defined the asymptotic average shadowing property for iterated function systems (IFS) and show that if iterated function system $\mathbb{F}$ has the asymptotic average shadowing property then $\mathbb{F}^k$ has the asymptotic average shadowing property for all $k \geq 0$. Also, if $\mathbb{F}$ is an IFS with the asymptotic average shadowing property (on $\mathbb{Z}_+$), then so $\mathbb{F}^{-1}$ has the asymptotic average shadowing property.

Keywords: Shadowing property, Asymptotic average shadowing property, Asymptotic-average pseudo orbit, IFS.

1 Introduction

The notion of shadowing is an important tool for studying properties of discrete dynamical systems. From numerical point of view, if dynamical system has the shadowing property, then numerically obtained orbits reflect the real behavior of trajectories of the systems. [1, 4]

Iterated function systems (IFS), are used for the construction of deterministic fractals and have found numerous applications, in particular to image compression and image processing. Important notions in dynamics like attractors, minimality, and shadowing can be extended to IFS. [2, 3, 6, 7]

Let (X, d) be a complete metric space. Let us recall that a parametrized Iterated Function system (IFS) $\mathbb{F} = \{X; f_\lambda | \lambda \in \Lambda\}$ is any family of continuous mappings $f_\lambda : X \rightarrow X, \lambda \in \Lambda$ where Λ is a finite nonempty set. [5]

Let $T = \mathbb{Z}$ or $T = \mathbb{Z}_+ = \{n \in \mathbb{Z} : n \geq 0\}$ and $\Lambda^{\mathbb{Z}_+}$ denote the set of all infinite sequences $\{\lambda_i\}_{i \in T}$ of symbols belonging to Λ . A typical element of $\Lambda^{\mathbb{Z}_+}$ can be denoted as $\sigma = \{\lambda_0, \lambda_1, \dots\}$ and we use the shorted notation

$$\mathbb{F}_{\sigma_n} = f_{\lambda_0} \circ f_{\lambda_1} \circ \dots \circ f_{\lambda_n}.$$

Definition 1.1. A sequence $\{x_n\}_{n \in T}$ is called an orbit of the IFS $\mathbb{F}$ if there exist $\sigma \in \Lambda^T$ such that $x_{n+1} = f_{\lambda_n}(x_n)$, for $\lambda_n \in \sigma$.

Given $\delta > 0$, a sequence $\{x_n\}_{n \in T}$ in X is called a δ -pseudo orbit of $\mathbb{F}$ if there exist $\sigma \in \Lambda^T$ such that for every $\lambda_n \in \sigma$, we have $d(x_{n+1}, f_{\lambda_n}(x_n)) < \delta$.

One says that the parameterized IFS has the shadowing property (on T), if given $\varepsilon > 0$,

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