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Talk G-ultrametric dynamics and some fixed point theorems pp.: 1–4

G-Ultrametric Dynamics and Some Fixed Point Theorems

Hamid Mamghaderi*
K. N. Toosi University of Technology

Hashem Parvaneh Masiha
K. N. Toosi University of Technology

Abstract

This paper is concerned with dynamics in general G-ultrametric spaces, hence we discuss the introduced concepts of these spaces. Also, the fixed point existing results of strictly contractive and non-expansive mappings defined on these spaces by inspiring from the theorem proved by Mustafa and Sims.

Keywords: Fixed point, G-ultrametric space, strictly contractive mapping, non-expansive mapping.
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1 Introduction

In 2005,Mustafa and Sims introduced a new class of generalized metric spaces (see [4, 5]), which are called G-metric spaces, as generalization of a metric space (X, d). Subsequently, many fixed point results on such spaces appeared (see, for example, [3, 1, 2]). Here, we present the necessary definitions and results in G-metric spaces, which will be useful for the rest of the paper. However, for more details, we refer to [4, 5].

Definition 1.1. [5]. Let X be a nonempty set. Suppose that $G : X \times X \times X \to [0, \infty)$ is a function satisfying the following conditions:

G1) $G(x, y, z) = 0$ if $x = y = z$;

G2) $0 < G(x, x, y)$, for all $x, y, z \in X$ with $x \neq y$;

G3) $G(x, x, y) \leq G(x, y, z)$; for all $x, y, z \in X$ with $z \neq y$;

G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \ldots$, (symmetry in all three variables), and

G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z \in X$, (rectangle inequality),

then the function G is called a generalized metric, or more specifically a G-metric on X, and the pair (X, G) is a G-metric space.

Definition 1.2. [5] Let (X, G) be a G-metric space, then for $x_0 \in X, r > 0$, the G-ball(dressed ball) with center x_0 and radius r is

$$B(x_0, r) = \{y \in X : G(x_0, y, y) < r\},$$

and the stripped ball of radius r and center x_0 is

$$B(x_0, r^+) = \{y \in X : G(x_0, y, y) \leq r\}$$

Proposition 1.3. *[5] Let (X, G) be a G-metric space, then for any $x_0 \in X$ and $r > 0$, we have,*

*Speaker