Vitality of Nodes in Networks Carrying Flows Over Time

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Abstract

In this paper finding most vital node of networks carrying flows over time is studied, a mathematical model is generalized and a fully combinatorial algorithm is provided adapting an iterative procedure. Given a network and a time horizon T, Most Vital Node Over Time (MVNOT) problem seeks for a node whose removal from network results greatest decrease in the value of maximum flow over time up to time horizon T between two terminal nodes.

Keywords: most vital nodes; maximum flow over time; combinatorial algorithm.
Mathematics Subject Classification [2010]: 90C11

1 Introduction

Vitality problem on networks is firstly introduced by Wolmer [4] at 1963. Wolmer [4] studied looking for a link whose removal from network results greatest decrease in the value of deterministic maximum flow between two predefined nodes. Later, many extensions of the original problem is studied in literature [2]. Recently, a new version of most vital link problem is introduced and studied by Morowati and Mehri [2] which differs from traditional models in the sense that it studies vitality on networks carrying flows over time [3] instead of traditional static flows.

In this paper we study the problem of finding most vital node of a network which aims to transfer maximum flow over time between two terminal nodes up to a predefined time horizon T. The MVNOT problem may simply be reduced to a most vital link problem but this reduction increase problem size significantly. Therefore, providing a direct solution method motivated us to provide an iterative algorithm for MVNOT problem.

2 Preliminaries

Let $G = (N, A, \mathbf{u}, \boldsymbol{\tau}, s, t)$ is given, where N is the set of nodes, A is the set of directed links with a positive capacity $\mathbf{u} = (u_{ij})_{(i,j)\in A}$ and positive transit times $\boldsymbol{\tau} = (\tau_{ij})_{(i,j)\in A}$, s is source node and t is terminal node. A static s-t-flow is a real valued mapping $\mathbf{x}$ on the links of G that satisfies capacity constraints $0 \le x_{ij} \le u_{ij}$ for all $(i,j) \in A$ and flow conservation constraints $\sum_{j\in N:(j,i)\in A} x_{ji} - \sum_{j\in N:(i,j)\in A} x_{ij} = 0$, for all $i \in N \setminus \{s,t\}$. The value of a static s-t-flow $\mathbf{x}$ is equal to $|\mathbf{x}| = \sum_{j\in N:(j,t)\in A} x_{jt} - \sum_{j\in N:(t,j)\in A} x_{tj}$.

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