Numerical solution for nth order linear Fredholm integro-differential equations by using Chebyshev wavelets integration operational matrix

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Abstract

In this paper, a numerical method for solving nth order linear Fredholm integro-differential equations is proposed. Proposed method is based on using Chebyshev wavelets integration operational matrix (CWIOM). Numerical tests to illustrate applicability of the new approach are presented.

Keywords: Fredholm integro-differential equations, Chebyshev wavelets, Operational matrix.
Mathematics Subject Classification [2010]: 13D45,39B42

1 Introduction

In recent years, numerical solution of integral equations and integro-differential equations by using Haar wavelets, Chebyshev wavelets, Legendre wavelets, CAS wavelets and other hybrid functions based on wavelets via integration operational matrix was discussed by many authors [1, 2, 3, 4]. Here, we consider the following nth order linear Fredholm integro-differential equation

$$\begin{cases} y^{(n)}(x) = f(x) + y(x) + \int_0^1 k(x,t)\left(y^{(n-1)}(t) + y^{(n-2)}(t) + \cdots + y'(t) + y(t)\right) dt \\ y(0) = y_0,\ y'(0) = y_1,\ y''(0) = y_2, \cdots, y^{(n-1)}(0) = y_{n-1}, \end{cases} \tag{1}$$

and proposed a new method based on CWIOM. In [1], the authors a numerical method based on for solving linear Fredholm integro-differential equation as

$$\begin{cases} y^{(n)}(x) = f(x) + y(x) + \int_0^1 k(x,t)y(t)dt \\ y(0) = y_0,\ y'(0) = y_1,\ y''(0) = y_2, \cdots, y^{(n-1)}(0) = y_{n-1}, \end{cases} \tag{2}$$

The main advantage of the proposed method in this paper is that in this method by using CWIOM and without any need to integration, we obtain the approximate solution of equation (1). The paper is organized as follows: In Sections 2 and 3, we recall properties of Chebyshev wavelets, function approximation and the operational matrix, respectively. In Section 4, the proposed method is applied to solve of the nth order linear Fredholm integro-differential equations. Some numerical examples are presented in Section 5.

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