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A note on an ideal of C(X) with λ- compact support

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Abstract

We introduce and investigate some properties of the set of functions in $C(X)$ with λ-compact support which is denoted by $C_K^\lambda(X)$, where λ is an infinite regular cardinal number. We extend some of the basic results concerning $C_K(X)$ (i.e., the family of all elements of $C(X)$ having compact support) for $C_K^\lambda(X)$. For instance, the purity of $C_K^\lambda(X)$ is studied and characterized through P_λ-spaces and λ-locally compact spaces which are not λ-compact. Finally some relations between topological properties of the space X and algebraic properties of the ideal $C_K^\lambda(X)$ are investigated.

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1 Introduction

Let $C(X)$ be the ring of all continuous real-valued functions on a completely regular Hausdorff space X. Throughout this article ideals are assumed to be proper ideals. For each $f \in C(X)$, let $Z(f) = \{x \in X : f(x) = 0\}$ and $cozf = X \setminus Z(f)$. If I is an ideal of $C(X)$, we put $cozI = \bigcup_{f\in I} cozf$. The support of f is the closure of $X \setminus Z(f)$ and $C_K(X)$ is the set of functions in $C(X)$ with compact support, see [4]. The concept λ-compact in [5] and [7], motivates us to introduce $C_K^\lambda(X)$. Our main purpose in this article is the study of the ideal structure of $C_K^\lambda(X)$ and of the relation between topological properties of the subspaces of X and algebraic properties of the ideal $C_K^\lambda(X)$. The space X is called λ-compact whenever each open cover of X can be reduced to an open cover of X whose cardinality is less than λ, where λ is the least infinite cardinal number with this property. We remind that the space X is P_λ-spaces if and only if every intersection with cardinality less than λ of open sets (i.e., G_λ-set) be open. The space X is called λ-locally compact space whenever every element of X has a λ-compact neighborhood, see [7]. For undefined terms and notations the reader is referred to [3] and [4].

2 Functions in $C(X)$ with λ-compact support

We need the following definition.

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