Sobolev Embedding theorem for weighted variable exponent Lebesgue space

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Abstract

This paper gives some Sobolev type embedding theorems for generalized weighted Lebesgue- Sobolev space $W^{1,p(x)}_{a(x)}(\Omega)$ where Ω is an open subset of $\mathbb{R}^N$ $(N \geq 2)$ with $p \in C(\overline{\Omega})$ and $a(x)$ is a measurable, nonnegative real valued function. The main result can be stated as follows, under some conditions we show the compact Sobolev embedding

$$W^{1,p(x)}_{a(x)}(\Omega) \hookrightarrow\hookrightarrow L^{q(x)}_{b(x)}(\Omega).$$

Keywords: variable exponent Lebesgue space, variable exponent Sobolev space, compact embedding.
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1 Introduction

The Sobolev space $W^{m,p}(\Omega)$, where p is constant, is suitable for studding of many problems in physics and mechanics. Whereas, by introducing the problems with p(x)- growth conditions that arising by studding some materials with inhomogeneities such as Electrorheological fluids, which was due to Willis Winslow in 1949, the classical Sobolev spaces do not work and so the variable exponent Lebesgue space $L^{p(\cdot)}(\Omega)$ and Sobolev space $W^{m,p(\cdot)}(\Omega)$ are defined, where $p(.)$ is some appropriate function; [7]. Despite the sufficient reasons for developing the Lebesgue and so the Sobolev space, the variable exponent Lebesgue and Sobolev spaces can be seen as a mathematical generalization of the classical space which are with constant exponent.

Hence the considerable attentions of mathematicians be involved in problems with p(x) growth conditions since the idea of generalizing the results has always been the incentive factor in Development of mathematics. We refer to [1] for the basic information about variable exponent Lebesgue and Sobolev spaces. Let Ω be an open subset of $\mathbb{R}^N$, $p \in L^\infty(\Omega)$ and

$$p^- := ess \inf_{x\in\Omega} p(x) \geq 1.$$

Moreover $a(x)$ is a measurable, nonnegative real valued function for $x \in \Omega$. The variable exponent Lebesgue space $\mathbf{L}^{p(\cdot)}(\Omega)$ is defined by

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