



Roman k -Domination Number Upon Vertex and Edge Removal

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Abstract

Let $k \geq 1$ be an integer. A *Roman k -dominating function* on a graph G with vertex set V is a function $f : V \rightarrow \{0, 1, 2\}$ such that every vertex $v \in V$ with $f(v) = 0$ has at least k neighbors $u_1, u_2, \dots, u_k$ with $f(u_i) = 2$ for $i = 1, 2, \dots, k$. The weight of a Roman k -dominating function is the value $f(V) = \sum_{v \in V} f(v)$. The minimum weight of Roman k -dominating functions on a graph G is called the *Roman k -domination number*, denoted by $\gamma_{kR}(G)$. In this paper, we consider the effects of vertex and edge removal on the Roman k -domination number of a graph. Some of our results improve these one given by Kämmerling and Volkmann in [6] for the Roman k -domination number.

Keywords: Roman domination, Roman k -domination number, Roman k -dominating function.

Mathematics Subject Classification [2010]: 13D45, 39B42

1 Introduction

For terminology and notation on graph theory not given here, the reader is referred to [5, 10]. In this paper, G is a simple graph with *vertex set* $V = V(G)$ and *edge set* $E = E(G)$. The *order* $|V|$ and the *size* $|E|$ are denoted by $n = n(G)$ and $m = m(G)$. For disjoint subsets A and B of vertices we denote by $E(A, B)$ the set of edges between A and B . The *open* and *closed neighborhoods* of a vertex $v \in V$ are $N_G(v) = \{u \in V | uv \in E\}$ and $N_G[v] = N_G(v) \cup \{v\}$, respectively. Also the *open* and *closed neighborhoods* of a subset $S \subseteq V(G)$ are $N_G(S) = \cup_{v \in S} N_G(v)$ and $N_G[S] = N_G(S) \cup S$, respectively. The *degree* of a vertex $v \in V$ is $\deg_G(v) = |N_G(v)|$. The *minimum* and *maximum degree* of a graph G are denoted by $\delta(G)$ and $\Delta(G)$, respectively. For a subset $S \subseteq V(G)$, the *induced subgraph* $G[S]$ is the subgraph of G with the vertex set S and for two vertices $u, v \in S$, $uv \in E(G[S])$ if and only if $uv \in E(G)$. We write $K_{p,q}$ for the *complete bipartite graph* with bipartition X and Y such that $|X| = p$ and $|Y| = q$. If $\omega(G)$ is the number of components of G , then $c(G) = m - n + \omega(G)$ is the well-known *cyclomatic number* of G .

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