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Solving the Black-Scholes equation through a higher order compact finite difference method

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Abstract

In this paper a new compact finite difference (CFD) method for solving Black-Scholes equation is analyzed. Thise method leads to a system of linear equations involving tridiagonal matrices and the rate of convergence of the method is of order $O(k^2 + h^8)$ where k and h are the time and space step-sizes, respectively. Numerical results obtained by the proposed method are compared with the exact solution.

Keywords: Option pricing, Black-Scholes equation, compact finite difference scheme
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1 Introduction

The Black-Scholes model [4, 5] is a powerful tool for valuation of equity options. This model is used for finding prices of stocks. Analytical approach and Numerical techniques are two ways for solving the European options. In [2] Mellin transformation was used to solve this model. They required neither variable transformation nor solving diffusion equation. R. Company et. al. [3] solved the modified Black-Scholes equation pricing option with discrete dividend. They used a delta-defining sequence of generalized Dirac-Delta function and applied the Mellin transformation to obtain an integral formula. Finally, they approximated the solution by using a numerical quadrature approximation.
Our contribution in this paper is the use of a high-order CFD method [1] for the pricing of options under the standard Black-Scholes model.

2 Construction of the method

Consider following Black-Scholes equation

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0, \tag{1}$$

where S is the asset value, σ is the volatility and r is the *risk-free interest rate*. If we denote the current price of the underlying by S, then the payoffs at expiry, T, for a given exercise price, K, of European Calls and Puts is

$$C(S,T) = \max(S-K, 0), \qquad P(S,T) = \max(K-S, 0). \tag{2}$$

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