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A generalized Hermite-Hadamard type inequality for h-convex functions via fractional integral

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Abstract

An inequality of Hermite-Hadamard type for h-convex functions via Riemann-Liouville fractional integral is studied. Our results generalize and improve the results of other researchers.

Keywords: Hermite-Hadamard's inequality, h-convex function, Riemann-Liouville fractional integral.
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1 Introduction

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a function defined on the interval I of the real numbers and $a, b \in I$, with $a < b$. If f is a convex function, then the Hermite-Hadamard inequality holds:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\,dx \leq \frac{f(a)+f(b)}{2}. \tag{1}$$

Definition 1.1. [4] Let $h : J \subseteq \mathbb{R} \to \mathbb{R}$ be a positive function. We say that $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is h-convex function or that f belongs to the class $SX(h, I)$ if f is nonnegative and for all $x, y \in I$ and $\lambda \in (0, 1)$ we have

$$f(\lambda x + (1-\lambda)y) \leq h(\lambda) f(x) + h(1-\lambda) f(y).$$

Notice that the class of h-convex functions generalizes the class of convex functions for $h(x) = x$ for all x.

Definition 1.2. [2] Let $f \in L_1[a, b]$. The Riemann-Liouville integrals $\mathbb{J}^{\alpha}_{a^+} f$ and $\mathbb{J}^{\alpha}_{b^-} f$ of order $\alpha > 0$ with $a \geq 0$ are defined by

$$\mathbb{J}^{\alpha}_{a^+} f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1} f(t)\,dt, \qquad x > a$$

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