Regularized Sinc-Galerkin Method for Solving a Two-Dimensional Nonlinear Inverse Parabolic Problem

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Abstract

In this paper, Sinc-Galerkin method is used to solve a two-dimensional nonlinear inverse parabolic problem and a stable numerical solution is determined. To do this, the Levenberg-Marquardt method is applied to deal with the ill-posedness of the discretized system. The accuracy and reliability of the proposed method is demonstrated by a test problem.

Keywords: Sinc-Galerkin method, Inverse parabolic problem, Levenberg-Marquardt method.
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1 Introduction

In this paper, a two-dimensional nonlinear inverse parabolic problem of the form

$$\begin{aligned} u_t - \Delta u = G(x,y,t,u), &\quad \text{(x,y)} \in \Omega \subset \mathbb{R}^2,\ t > 0,\ n > 1,\ (n \in \mathbb{N}), \\ u(x,y,0) = 0, &\quad \text{(x,y)} \in \Omega \subset \mathbb{R}^2, \\ u(x,y,t) = 0, &\quad \text{(x,y)} \in \partial\Omega \subset \mathbb{R}^2,\ t \geqslant 0, \end{aligned} \tag{1}$$

is considered, where $\partial\Omega$ is the boundary of $\Omega = [0,1] \times [0,1]$, $G(x,y,t,u) = f(x,y) + H(x,y,t) - u^n$ such that $H(x,y,t)$ is known a function and the functions $f(x,y)$ and $u(x,y,t)$ are unknown. If $f = f(x,y)$ is given, then the problem (1) is called the *direct problem* (DP). The existence and uniqueness of the DP (1) have been investigated in [1]. To find the pair (u,f), we use the overposed measured data

$$u(x^*,y^*,t_i) = E(t_i), \qquad 0 < x^*,\ y^* < 1,\ i = 1,2,\dots,I. \tag{2}$$

Let us denote by the notation $u[x,y,t;f]$ the solution of the DP (1). Then from the additional condition (2) it is seen that the nonlinear inverse parabolic problem (1) consists of solving the following nonlinear functional equation

$$u[x^*,y^*,t_i;f] = E(t_i), \qquad 0 < x^*,\ y^* < 1,\ i = 1,2,\dots,I. \tag{3}$$

In general, instead of solving the functional equation (3), an optimization problem is solved, where objective function is minimized by an effective regularization method. This objective function is defined by

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