Shifted Legendre pseudospectral approach for solving population projection models

Ali Najafi Abrand Abadi*
Department of Humanities and Social Sciences, Science and Research Branch,
Islamic Azad University, Tehran, Iran
Habib Allah Zanjani
Department of Humanities and Social Sciences, Science and Research Branch,
Islamic Azad University, Tehran, Iran

Abstract

In this investigation, a numerical technique based on shifted Legendre polynomials for solving population projection models is proposed. The approach reduces the solution of the main problem to the solution of a system of nonlinear algebraic equations. The comparison of the results with the analytical and numerical solution show the efficiency and accuracy of presented method.

Keywords: Population projection models, Logistic growth model, Pseudospectral method, Shifted Legendre polynomials
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1 Introduction

Population dynamics has traditionally been the dominant branch of mathematical biology, whose history spans more than 200 years [1, 5]. A projection may be defined as the numerical outcome of a particular set of assumptions regarding the future population [1]. Most mathematical models that describe the dynamics of a population over time $u(t)$ are based on first order differential equation of the form:

$$u'(t) = Au(t) - u(t)F(u(t)) + B, \quad u(0) = \beta, \quad t \geq 0. \tag{1}$$

In population models the solution $u(t)$ of (1) corresponds to the population density at time t, the linear term $Au(t)$ corresponds to intrinsic growth, loss, or transition processes in the population independent of population density. The nonlinear logistic term $-u(t)F(u(t))$ in (1) corresponds to loss processes due to crowding at a rate proportional to a functional of the population density. Lastly, the constant term B corresponds to an external source of population growth, independent of the population density.

*Speaker