Pseudonumerical range of matrices

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Abstract

In this paper for a given $\epsilon > 0$ and an $n \times n$ complex matrix A, the notion of pseudonumerical range of A is introduced. Also, some algebraic and geometrical properties of this notion are investigated moreover the relationship between this notion and the pseudospectrum of A is stated.

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1 Introduction

Let $\mathbb{M}_n(\mathbb{C})$ be the algebra of all $n \times n$ complex equipped with the operator norm $\|.\|$ induced by the usual vector norm $\|x\| = (x^*x)^{1/2}$ on $\mathbb{C}^n$, i.e.,

$$\|A\| = max\{\|Ax\| : x \in \mathbb{C}^n, \|x\| = 1\}.$$

In our discussion we assume that $D(a, r) = \{\mu \in \mathbb{C} : |\mu - a| < r\}$, where $a \in \mathbb{C}$ and $r > 0$. Also, we use the convention that if z is an eigenvalue of $A \in \mathbb{M}_n(\mathbb{C})$, then $\|(A - zI)^{-1}\| := \infty$. For $\epsilon > 0$ and a matrix $A \in \mathbb{M}_n(\mathbb{C})$, the pseudospectrum of A is defined and denoted, e.g., see [4], by

$$\sigma_\epsilon(A) = \{z \in \mathbb{C} : \|(A - zI)^{-1}\| > 1/\epsilon\}. \tag{1}$$

It is known that

$$\begin{aligned} \sigma_\epsilon(A) &= \{z \in \sigma(A + E) : E \in \mathbb{M}_n \text{ and } \|E\| < \epsilon\} \\ &= \{z \in \mathbb{C} : s_n(zI - A) < \epsilon\}, \end{aligned} \tag{2}$$

where $s_n(.)$ denotes the smallest singular value.
Pseudospectrum provides an analytical and graphical alternative for investigating nonnormal matrices and operators, gives a quantitative estimate of departure from non-normality

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