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A numerical scheme for two-dimensional optimal control problems with Grünwald-Letnikov for Riesz Fractional Derivatives

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Abstract

In this paper, we study control systems containing a Riesz fractional derivative and solve this problem by a numerical method which is so called Grünwald-Letnikov approximation scheme . A two-dimensional fractional optimal control problem is studied as an example to demonstrate the performance of this method.

Keywords: Calculus of variations, Riesz fractional derivative, Grünwald-Letnikov
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1 Introduction

Fractional calculus (FC) generalizes integrals and derivatives to non-integer orders. During the last decade, FC was found to play a fundamental role in the modeling of a considerable number of phenomena, in particular, the modeling of memory dependent phenomena and complex media such as porous media. first we define a fractional derivative, and then formulate a fractional optimal control problem (FOCPs) and find the necessary conditions for optimality. The left and right Riemann-Liouville fractional derivatives of order α are defined respectively:

$$ {}_aD_t^{\alpha}y(t) = \frac{1}{\Gamma(1-\alpha)}(\frac{d}{dt})\int_a^t (t-\tau)^{-\alpha}y(\tau)d\tau, $$
$$ {}_tD_b^{\alpha}y(t) = \frac{1}{\Gamma(1-\alpha)}(\frac{d}{dt})\int_t^b (\tau-t)^{-\alpha}y(\tau)d\tau, $$

where $n-1<\alpha<n$. The usual definitions of the derivatives are obtained when α is an integer. Note that for $\alpha \in (0,1)$, the fractional operators are non-local. One space needed Riesz fractional derivative ${}_a^RD_b^{\alpha}y(t)$ is given by

$$ {}_a^RD_b^{\alpha}y(t) = \frac{1}{2}({}_aD_t^{\alpha}y(t) -{}_t D_b^{\alpha}y(t)), $$
$$ {}_b^RD_t^{\alpha}y(t) = \frac{1}{2}({}_tD_b^{\alpha}y(t) -{}_a D_t^{\alpha}y(t)). \tag{1} $$

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