



## Estimation of local stresses and elastic properties of a mortar sample by FFT computation of fields on a 3D image

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### ABSTRACT

This study concerns the prediction of the elastic properties of a 3D mortar image, obtained by microtomography, using a combined image segmentation and numerical homogenization approach. The microstructure is obtained by segmentation of the 3D image into aggregates, voids and cement paste. Full-fields computations of the elastic response of mortar are undertaken using the Fast Fourier Transform method. Emphasis is made on highly-contrasted properties between aggregates and matrix, to anticipate needs for creep or damage computation. The representative volume element, i.e. the volume size necessary to compute the effective properties with a prescribed accuracy, is given. Overall, the volumes used in this work were sufficient to estimate the effective response of mortar with a precision of 5%, 6% and 10% for contrast ratios of 100, 1000 and 10,000, resp. Finally, a statistical and local characterization of the component of the stress field parallel to the applied loading is carried out.

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### 1. Introduction

The mechanical properties of cementitious and mortar-based composites strongly depend on the multi-scale distribution of fine aggregates and voids inside cement paste as well as on the shape and size of aggregates. Additionally, such materials usually exhibit a great diversity in the microstructure, as a consequence of their manufacturing process. Accordingly, a large number of experiments on real samples should be carried out to estimate precisely the properties of such composites. As a consequence, efforts have been made to simulate both the complex microstructures of such materials and their mechanical response, by numerical means. One of the main difficulties of the latter relates to large-size computations in homogenization. Memory and time-efficient numerical tools are required to predict the effective mechanical properties of microstructure samples. Microstructures are obtained either from a real material by segmentation of an image or from a random morphological simulation. Nagai et al. [16] used a 3D concrete microstructure reconstructed from 2D successive sections to investigate both linear elastic behavior and cracks along the interface between aggregates and matrix. Microtomography has been used by Hain et al. [9] to study linear elastic and viscoplastic behaviors of hardened cement paste by the finite element method (FEM). Various random models of cementitious materials have been developed as well. For mortar and cement materials, the model of Bentz [2], which takes into account

hydration process, has been used by Hain et al. [9], Haecker et al. [7] and Bernard et al. [3] to investigate linear elasticity of cement paste [7,9] and of mortar [3]. Šmilauer et al. [21] also used Bentz model combined with Fast-Fourier Transform method to study viscoelastic behavior of cement paste. They considered random microstructures of various sizes, the largest containing  $200 \times 200 \times 200$  voxels, and low Young modulus contrast between matrix and aggregates. Bary et al. [1] modeled cement paste as inclusions and pores embedded in the C–S–H matrix. Two models were considered for the particles: spheres or a mix of inclusions with spherical and prismatic shapes. The linear elastic response is evaluated thanks to FEM computations, for various types of boundary conditions. The effective properties are compared with the Mori-Tanaka and self-consistent analytical theories. Wriggers et al. [24] proposed a random model of concrete made of spherical aggregates in a mortar matrix. The model assumes isotropy of the material and a minimal distance between particles, which depends on their volume fraction. The linear elastic response, as well as damage degradation, is investigated numerically usually with FEM. Two models of microstructures were also proposed by Häfner et al. [8] and Caballero et al. [5] and used to investigate the mechanical properties of concrete. In Häfner's model, the aggregates' shape is a modified ellipsoid with a sine function added, with linear elasticity investigated with a multigrid method. Caballero et al. [5] modeled concrete as large aggregates embedded in a matrix representing mortar and fine aggregates. Larger particles were generated as polyhedra obtained by Voronoï tessellation. An interface transition zone was taken into account with zero thickness element to investigate cracking. This model was used to compute numerically the response of mortar to a uniaxial tension test.

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