# Fixed Point Theorems for $G$-Nonexpansive Mappings in Ultrametric Spaces and non-Archimedean Normed Spaces Endowed with a Graph

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**Abstract**

The purpose of this article is to present some new fixed point results for $G$-nonexpansive mappings defined on an ultrametric space and non-Archimedean normed space which are endowed with a graph. In particular, we investigate the relationship between weak connectivity and the existence of fixed points for these mappings.

**Keywords:** Fixed point; ultrametric space; spherically complete ultrametric spaces; non-Archimedean space: nonexpansive mapping.
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## 1 Introduction

Let $(X, d)$ be a metric space. $(X, d)$ is called an ultrametric space if the metric $d$ satisfies the strong triangle inequality, i.e., for all $x, y, z \in X$:

$$d(x, y) \leq \max\{d(x, z), d(y, z)\},$$

in this case $d$ is said to be ultrametric [4].

We denote by $B(x, r)$, the closed ball $B(x, r) = \{y \in X : d(x, y) \leq r\}$, where $x \in X$ and $r \geq 0$ ($B(x, 0) = \{x\}$). A known characteristic property of ultrametric spaces is the following:

If $x, y \in X$, $0 \leq r \leq s$ and $B(x, r) \cap B(y, s) \neq \emptyset$, then $B(x, r) \subset B(y, s)$.

An ultrametric space $(X, d)$ is said to be spherically complete if every shrinking collection of balls in $X$ has a nonempty intersection [4]. [4] Let $\mathbb{K}$ be a non-Archimedean valued field. A norm on a vector space $X$ over $\mathbb{K}$ is a map $\| \cdot \|$ from $X$ into $[0, \infty)$ with the following properties:

1) $\|x\| \neq 0$ if $x \in E \setminus \{0\}$;

2) $\|x + y\| \leq \max\{\|x\|, \|y\|\} \qquad (x, y \in X)$;

3) $\|\alpha x\| = |\alpha| \|x\| \qquad (\alpha \in \mathbb{K}, x \in X)$.

In 1993, Petalas and Vidalis [3] proved the following theorem:

**Theorem 1.1** ([3]). *Let $X$ be a non-Archimedean spherically complete normed space. If $T : X \longrightarrow X$ is a nonexpansive mapping, i.e., $d(Tx, Ty) \leq d(x, y) \quad x, y \in X$, Then either $T$ has at least one fixed point or there exists a $T$-invariant closed ball $B$ with radius $r > 0$ such that $\|b - Tb\| = r$ for all $b \in B$.*

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