Some Fixed Point Theorems For Mappings on a G-Metric Space Endowed with a Graph

Nahid Sadat Mohseni*
K. N. Toosi University of Technology

Hashem Parvaneh Masiha
K. N. Toosi University of Technology

Abstract

In this paper, we discuss the existence of fixed points for Banach and Kannan contractions defined on G-metric spaces, which were introduced by Mustafa and Sims, endowed with a graph. Our results generalize and unify some recent results by Jachymski, Bojor and Mustafa and those contained therein. Moreover, we provide some examples to show that our results are substantial improvement of some known results in literature.

Keywords: Fixed point, G-metric spaces, Banach contraction, Kannan contraction.
Mathematics Subject Classification [2010]: 47H10, 47H09.

1 Introduction

Investigation of the existence and uniqueness of fixed points of certain mappings in the framework of metric spaces is one of the centers of interests in nonlinear functional analysis. Fixed point theory has a wide application in almost all fields of quantitative sciences such as economics, biology, physics, chemistry, computer science and many branches of engineering. It is quite natural to consider various generalizations of metric spaces in order to address the needs of these quantitative sciences. Different mathematicians tried to generalize the usual notion of metric space (X, d). In the 1960s, Gähler [4] tried to generalize the notion of metric and introduced the concept of 2-metric spaces inspired by the mapping that associated the area of a triangle to its three vertices. But different authors proved that there is no relation between these two functions [5]. Then, Dhage [3] in his Ph. D. thesis introduce a new class of generalized metric space called D-metric spaces. Unfortunately, both kinds of metrics appear not to have as good properties as their authors announced ([5] ,[8]). To overcome these drawbacks, in 2003 Mustafa and Sims [7] showed that most of the results claimed concerning of such spaces are invalid. Then they introduced a generalization of metric spaces (X, d), which are called G-metric spaces ([8], [9]). The G-metric space is defined as follows:

Definition 1.1 ([9]). Let X be a nonempty set, and $G : X \times X \times X \to [0, +\infty)$ be a function satisfying:

(G1) $G(x, y, z) = 0$ if $x = y = z$,
(G2) $0 < G(x, x, y)$; for all $x, y \in X$, with $x \neq y$,
(G3) $G(x, x, y) \leq G(x, y, z)$; for all $x, y, z \in X$ with $z \neq y$,
(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = ...$, (symmetry in all three variables),
(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z, a \in X$, (rectangle inequality).

Then the function G is called a generalized metric, or, more specifically a G-metric on X, and the pair (X, G) is a G-metric spaces.

A G-metric space (X, G) is called symmetric G-metric space if $G(x, x, y) = G(x, y, y)$; for all $x, y \in X$.

*Speaker