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Lie Ternary (σ, τ, ξ)-Derivations on Banach Ternary Algebras

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Abstract

Let A be a Banach ternary algebra over a scalar field $\mathbb{R}$ or $\mathbb{C}$ and X be a Banach ternary A-module. Let σ, τ and ξ be linear mappings on A. We define a Lie ternary (σ, τ, ξ)-derivation. Moreover, we prove the generalized Hyers-Ulam-Rassias stability of lie ternary (σ, τ, ξ)-derivations on Banach ternary algebras.

Keywords: Banach ternary A-module, Lie ternary (σ, τ, ξ)-derivation, Hyers–Ulam–Rassias stability.

1 Introduction

Let A be a Banach ternary algebra and X be a Banach space. Then X is called a ternary Banach A-module, if module operations $A \times A \times X \to X$, $A \times X \times A \to X$, and $X \times A \times A \to X$ are $\mathbb{C}$-linear in every variable. Moreover satisfy:

$$\max\{\|[xab]_X\|, \|[axb]_X\|, \|[abx]_X\|\} \leq \|a\|\|b\|\|x\|$$

for all $x \in X$ and all $a, b \in A$.

The stability of functional equations was started in 1940 with a problem raised by S. M. Ulam [6]. In 1941 Hyers affirmatively solved the problem of S. M. Ulam in the context of Banach spaces. In 1950 T.Aoki [2] extended the Hyers' theorem. in 1978, Th. M. Rassias [5] formulated and proved the following Theorem:
Assume that E_1 and E_2 are real normed spaces with E_2 complete, $f : E_1 \to E_2$ is a mapping such that for each fixed $x \in E_1$ the mapping $t \to f(tx)$ is continuous on $\mathbb{R}$, and let there exist $\epsilon \geq 0$ and $p \in [0, 1)$ such that $\|f(x + y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p)$ for all $x, y \in E_1$. Then there exists a unique linear mapping $T : E_1 \in E_2$ such that $\|f(x) - T(x)\| \leq \epsilon \frac{\|x\|^p}{(1-2^p)}$ for all $x \in E_1$.
The equality $\|f(x + y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p)$ has provided extensive influence in the development of of what we now call Hyers-Ulam-Rassias stability of functional equations [3]. In 1994, a generalization of Rassias' theorem was obtained by Gavruta [4], in which he replaced the bound $\epsilon(\|x\|^p + \|y\|^p)$ by a general control function.

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