# Some new singular value inequalities for compact operators

Ali Taghavi
University of Mazandaran

Vahid Darvish*
University of Mazandaran

**Abstract**

In this paper, by applying the concept of operator $h$-convex functions we prove several singular value inequalities for operators which provide refinements of previous results.

**Keywords:** Hermite-Hadamard inequality, Operator $h$-convex function, Singular value inequality
**Mathematics Subject Classification [2010]:** 47A63, 47B05, 26D15

## 1 Introduction

Let $B(H)$ stand for the $C^*$-algebra of all bounded linear operators on a complex separable Hilbert space $H$ with inner product $\langle \cdot, \cdot \rangle$. An operator $A \in B(H)$ is positive and write $A \geq 0$ if $\langle Ax, x\rangle \geq 0$ for all $x \in H$. Let $B(H)^+$ stand for all positive operators in $B(H)$.

If $A$ is a self-adjoint operator and $f$ is a real valued continuous function on $\mathrm{Sp}(A)$, then $f(t) \geq 0$ for any $t \in \mathrm{Sp}(A)$ implies that $f(A) \geq 0$.

The following inequality holds for any convex function $f$ defined on $\mathbb{R}$

$$(b-a)f\left(\frac{a+b}{2}\right) \leq \int_a^b f(x)dx \leq (b-a)\frac{f(a)+f(b)}{2}, \quad a, b \in \mathbb{R}. \tag{1}$$

A real valued continuous function $f$ on an interval $I$ is said to be *operator convex* if

$$f((1-\lambda)A + \lambda B) \leq (1-\lambda)f(A) + \lambda f(B), \tag{2}$$

in the operator order, for all $\lambda \in [0, 1]$ and for every self-adjoint operator $A$ and $B$ on a Hilbert space $H$ whose spectra are contained in $I$ (see [3]).

As an example of such functions, we note that $f(t) = t^r$ is operator convex on $(0, \infty)$ if either $1 \leq r \leq 2$ or $-1 \leq r \leq 0$ and is operator concave on $(0, \infty)$ if $0 \leq r \leq 1$ (see [1, p.147]).

In [3], Dragomir investigated the operator version of the Hermite-Hadamard inequality for operator convex functions asserts that if $f : I \to \mathbb{R}$ is an operator convex function on the interval $I$ then, for any self-adjoint operators $A$ and $B$ with spectra in $I$ the following inequalities hold

$$f\left(\frac{A+B}{2}\right) \leq \int_0^1 f\left((1-t)A + tB\right) dt \leq \frac{f(A)+f(B)}{2}. \tag{3}$$

*Speaker