Amenability of Vector Valued Group algebras

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Abstract

Generalizing the notion of amenability for $L^1(G)$, we study the concept of amenability of $L^1(G, A)$. Among the other things, we prove that $L^1(G, A)$ is approximately weakly amenable where A is a unital separable Banach algebra. We investigate the existence of a left invariant mean on various vector valued function spaces. The candidates for the choice of space are $LUC(G, A^*)$, $WAP(G, A^*)$ and $C_0(G, A^*)$.

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1 Introduction

It is a well-known theorem of Johnson that a locally compact group G is amenable if and only if $L^1(G)$ is amenable. We now switch from groups to vector-valued Banach algebras. Our references for vector-valued integration theory is [1], [2]. Let G be a locally compact group with a fixed left Haar measure m and A be a unital separable Banach algebra. Let $L^1(G, A)$ be the set of all measurable vector-valued (equivalence classes of) functions $f : G \to A$ such that $\|f\|_1 = \int_G \|f(t)\|dm(t) < \infty$. Equipped with the norm $\|.\|_1$ and the convolution product * specified by

$$f * g(x) = \int f(t)g(t^{-1}x)dm(t) \ (f, g \in L^1(G, A)),$$

$L^1(G, A)$ is a Banach algebra. It is our objective in this paper to demonstrate the corresponding characterization of $L^1(G, A)$. $M(G, A)$ will denote the space of regular A-valued Borel measures of bounded variation on G. $L^1(G, A)$ is a closed two-sided ideal of $M(G, A)$.

Another space considered in this paper is $L^\infty(G, A^*)$, which consists of maps f of G into A^* that are scalarwise measurable and $N_\infty(\|f\|) = \text{loc ess sup}_{t\in G}(\|f(t)\|) < \infty$. The dual of $L^1(G, A)$ may be identified with $L^\infty(G, A^*)$ [2]. We show that every continuous derivation from $L^1(G, A)$ into $L^\infty(G, A^*)$ is approximately inner, that is, of the form

$$D(a) = \lim_\alpha (F_\alpha.a - a.F_\alpha)$$

for some $\{F_\alpha\}_{\alpha\in I} \in L^\infty(G, A^*)$.

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