Numerical solution of the time fractional Fokker-Planck equation using local discontinuous Galerkin method

Jafar Eshaghi*
Amirkabir University of Technology

Hojatollah Adibi
Amirkabir University of Technology

Abstract

In this article, we will offer the numerical solutions of time fractional Fokker-Planck equations (TFFPE). Two methods for discretization in time variable are investigated. The first method is based on a fractional finite difference scheme (FFDS) and in the second method the time fractional derivative is replaced by the Volterra integral equation which could be computed by the trapezoidal quadrature scheme (TQS). Then we have applied the local discontinuous Galerkin method in space for both methods. Some linear and nonlinear test problems have been considered to show the validity and convergence of two proposed methods. The results show that FFDS and TQS are of $2-\alpha$ and second–order accurate in time variable, respectively.

Keywords: Time fractional Fokker-Planck equation; discontinuous Galerkin method.
Mathematics Subject Classification [2010]: 45D05; 45G05; 41A30.

1 Introduction

Fractional calculus have a long history, having been mentioned by Leibniz in a letter to L'Hospital in 1695.
This paper mainly focuses on a numerical algorithm for finding the approximate solution of the nonlinear fractional Fokker–Planck equations with time–fractional derivative of the form:

$$\frac{\partial^\alpha u}{\partial t^\alpha} = [-\frac{\partial}{\partial x}A(x,t,u) + \frac{\partial^2}{\partial x^2}B(x,t,u)]u(x,t), \quad t>0, \quad \alpha>0, \tag{1}$$

2 Main results

In this section we give some basic definitions and properties of the fractional calculus theory which are needed next.
Definition 2.1. The Caputo derivative is defined as follows:

$$D_*^\alpha f(x) \quad = \quad \frac{1}{\Gamma(n-\alpha)}\int_0^x (x-t)^{n-\alpha-1}\frac{d^n f(t)}{dx^n}dt, \quad \alpha\in(n-1,n], n\in N,$$

*Speaker