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Decomposing modules into modules with local endomorphism rings

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Abstract

Let R be a right artinian ring or a perfect commutative ring. Let M be a noncosingular lifting module that does not have relatively projection component. Then $M = \oplus_{i=1}^{n} M_i$ has the exchange property and the decomposition complements direct summands, where each endomorphism ring $End(M_i)$ is local.

Keywords: noncosingular module; lifting module; local endomorphism ring.
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1 Introduction

Throughout this paper R will denote an associative ring with identity. Modules over R will be right R-modules. We will use the notation $N \ll M$ to indicate that N is small in M (i.e. $\forall L \lneq M, L+N \neq M$). $Rad(M)$ will denote the Jacobson radical of M. A non-zero module M is called *hollow* if every proper submodule of M is small in M. M is called *local* if the sum of all proper submodules of M is also a proper submodule of M. It is clear that every local module is hollow. A module M is called *lifting* if for every submodule $A \leq M$, there exists a direct summand B of M such that $B \leq A$ and $A/B \ll M/B$. Lifting modules are dual notions of extending modules and [3] deals with different aspects of lifting modules. A module M is amply supplemented and every coclosed submodule of M is a direct summand of M if and only if M is lifting by [3, 22.3(d)]. In [5] Talebi and Vanaja defined $\overline{Z}(M)$ as follows:

$$\overline{Z}(M) = \mathrm{Re}\,(\mathrm{M}, \mathcal{S}) = \bigcap \{\mathrm{Ker(g)} \mid \mathrm{g} \in \mathrm{Hom}\,(\mathrm{M}, \mathrm{L}), \mathrm{L} \in \mathcal{S}\},$$

where $\mathcal{S}$ denotes the class of all small modules. They called M a *cosingular* (*noncosingular*) module if $\overline{Z}(M) = 0$ ($\overline{Z}(M) = M$).

A family $\{X_\lambda : \lambda \in \Lambda\}$ of submodules of a module M is called a *local summand* of M, if $\sum_{\lambda\in\Lambda} X_\lambda$ is direct and $\sum_{\lambda\in F} X_\lambda$ is a summand of M for every finite subset $F \subseteq \Lambda$. If even $\sum_{\lambda\in\Lambda} X_\lambda$ is a summand of M, we say that *the local summand is a summand.* A module M is said to have the *(finite) exchange property* if for any (finite) index set I, whenever $M \oplus N = \oplus_{i\in I} A_i$ for modules N and A_i, then $M \oplus N = M \oplus (\oplus_{i\in I} B_i)$ for submodules $B_i \leq A_i$. Let $M = \oplus_I M_i$ be a decomposition of the module M into nonzero summands M_i. This decomposition is said to *complement direct summands* if, whenever A is a direct summand of M, there is a subset J of I for which $M = (\oplus_J M_j) \oplus A$.