AN ITERATIVE METHOD FOR NONEXPANSIVE MAPPINGS IN HILBERT SPACES

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Abstract

In this paper, with a different iterative method for finding a common fixed point of a countable nonexpansive mappings a strong convergence theorem for a countable family of nonexpansive mappings in a Hilbert space is given. This theorem complete some recent results.

Keywords: Fixed points; Nonexpansive mapping; Iterative method; Variational inequality; Hilbert space.
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1 Introduction

Moudafi introduced the viscosity approximation method for nonexpansive mappings. Let f be a contraction on H, starting with an arbitrary initial $x_0 \in H$, define a sequence $\{x_n\}$ recursively by

$$x_{n+1} = \alpha_n f(x_n) + (1 - \alpha_n)Tx_n, n \geq 0, \tag{1}$$

where $\{\alpha_n\}$ is a sequence in $(0, 1)$.
Xu proved that under certain appropriate conditions on $\{\alpha_n\}$, the sequence $\{x_n\}$ generated by (1) converges strongly to the unique solution x^* in $Fix(T)$ of the variational inequality:

$$\langle (I - f)x^*, x^* - x\rangle \leq 0, \forall x \in Fix(T). \tag{2}$$

We know iterative methods for nonexpansive mappings can be used to solve a convex minimization problem. See, e.g., [4, 5] and references therein. A typical problem is that of minimizing a quadratic function on the set of the fixed points of nonexpansive mapping on a real Hilbert space

$$\min_{x\in Fix(T)} \frac{1}{2}\langle Ax, x\rangle - \langle x, a\rangle, \tag{3}$$

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