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Real interpolation of quasi-Banach spaces

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Abstract

We inter relate the real interpolation space with the quasi-Banach couple (A_0, A_1), (A_0+A_1, A_1) and $(A_0, A_0\cap A_1)$ that A_j is c_j normed. Proving among others the identities

$$(A_0 + A_1, A_1)_{\theta,q} \cap A_0 = (A_0, A_1)_{\theta,q} \cap A_0 = (A_0, A_0 \cap A_1)_{\theta,q}.$$
$$(A_0 \cap A_1, A_1)_{\theta,q} + A_0 = (A_0, A_1)_{\theta,q} + A_0 = (A_0, A_0 + A_1)_{\theta,q}.$$

for all $0 < q \leq \infty$, $0 < \theta < 1$, and $c_1/c_0 \leq 1$.

Keywords: quasi-Banach spaces, interpolation space, real method of interpolation
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1 Introduction

Our main reference to the theory of interpolation space is [1]. Let $\bar{A} = (A_0, A_1)$ be a quasi-Banach couple, let $0 < \theta < 1$ and $0 < q \leq \infty$. The real interpolation space $(A_0, A_1)_{\theta,q}$ consist of all elements $a \in A_0 + A_1$ having a finite quasi-norm

$$\|a\|_{\theta,q,} = \begin{cases} (\sum_{\nu\in Z}(2^{-\nu\theta}K(2^\nu, a))^q)^{1/q} & if\ 0 < q < \infty \\ sup_{\nu\in Z}\{2^{-\nu\theta}K(2^\nu, a)\} & if\ q = \infty \end{cases}.$$

Here, for $0 < t < \infty$, we put

$$K(t, a) = K(t, a; A_0, A_1) = inf\{\|a_0\|_{A_0} + t\|a_1\|_{A_1}\ :\ a = a_0 + a_1, a_j \in A_j\}$$

and similarly the J-functional for $a \in A_0 \cap A_1 := \triangle(\bar{A})$ by

$$J(t, a; \bar{A}) = max\{\|a\|_{A_0}, t\|a\|_{A_1}\ :\ a \in \triangle(\bar{A})\}.$$

For $0 < \theta < 1$ we abbreviate $\bar{\theta} = max(\theta, 1 - \theta)$ and $\underline{\theta} = min(\theta, 1 - \theta)$.

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