46^{th} Annual Iranian Mathematics Conference
25-28 August 2015
Yazd University

Minimum size of intersetion for covering groups by subgroups

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Abstract

Let G denotes a semisimple $\mathfrak{C}_8$-group and $\{M_i \mid 1 \leq i \leq 8\}$ be a maximal irredundant 8-cover for G, with core-free intersection $D = \bigcap_{i=1}^{8} M_i$. Also for each i, $1 \leq i \leq 8$ we assume that $|G : M_i| = \alpha_i$ such that $\alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \alpha_4 \leq \alpha_5 \leq \alpha_6 \leq \alpha_7 \leq \alpha_8$.

Let l is minimum positive integer such that $\bigcap_{i=1}^{l}(M_i)_G \neq 1$. We say that l is minimum size of intersetion and in this case we show that MSI(G)=l. In this paper we show that if G be a semisimple $\mathfrak{C}_8$-group and $\alpha_l \leq 4$ then MSI(G)≤ 3

Keywords: covering groups by subgroups, Subdirect product, maximal iredundant cover, core-free intersection
Mathematics Subject Classification [2010]: 20F99

1 Introduction and history

Let G be a group. A set $\mathcal{C}$ of proper subgroups of G is called a cover for G if its set-theoretic union is equal to G. If the size of $\mathcal{C}$ is n, we call $\mathcal{C}$ an n-cover for the group G. A cover $\mathcal{C}$ for a group G is called irredundant if no proper subset of $\mathcal{C}$ is a cover for G. A cover $\mathcal{C}$ for a group G is called core-free if the intersection $D = \bigcap_{M \in \mathcal{C}} M$ of $\mathcal{C}$ is core-free in G, i.e. $D_G = \bigcap_{g \in G} g^{-1}Dg$ is the trivial subgroup of G. A cover $\mathcal{C}$ for a group G is called maximal if all the members of $\mathcal{C}$ are maximal subgroups of G. A cover $\mathcal{C}$ for a group G is called a $\mathfrak{C}_n$-cover whenever $\mathcal{C}$ is an irredundant maximal core-free n-cover for G and in this case we say that G is a $\mathfrak{C}_n$-group. A finite group is called semisimple if it has no non-trivial normal abelian subgroups (see p. 86 of [9] for further information on such groups).

Also we use the usual notations ([9]); for example, C_n denotes the cyclic group of order n, $(C_n)^j$ is the direct product of j copies of C_n, the core of a subgroup H of G is denoted by H_G.

In [10], Scorza determined the structure of all groups having an irredundant 3-cover with core-free intersection.

Theorem 1.1. (Scorza [10]) *Let $\{A_i : 1 \leq i \leq 3\}$ be an irredundant cover with core-free intersection D for a group G. Then $D = 1$ and $G \cong C_2 \times C_2$.*

In [7], Greco characterized all groups having an irredundant 4-cover with core-free intersection. Bryce et al.[6], characterized groups with maximal irredundant 5-cover with core-free intersection.

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