Some Properties Of n-almost Prime Submodules *

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Abstract

Prime ideals have many important properties and so its generalizations have been studied in many papers. The notion of n-almost prime submodules is generalization of prime submodules. In this article we study the behavior of n-almost Prime ideals in unique factorization domains and also we find some properties of n-almost Prime submodules of PI-multiplication modules.

Keywords: n-almost prime submodule, unique factorization domain, PI-multiplication modules

Mathematics Subject Classification [2010]: 13E05, 13C99, 13C13, 13F05, 13F15.

1 Introduction

Throughout this paper all rings are commutative with identity and all modules are unitary. Also we consider $n > 1$ a positive integer. Let N be a submodule of an $R-$module M. The set $\{r \in R | rM \subseteq N\}$ is denoted by $(N : M)$ and particularly we denote $\{r \in R | rN = 0\}$ by $ann(N)$. Also we consider $T(M) = \{m \in M | \exists 0 \neq r \in R, \ rm = 0\}$. A module M is called torsion, if $T(M) = M$. If $T(M) = 0$, it is said that M is a torsion-free module.

An *n-almost prime* ideal was introduced in [1]. The concept of n-almost prime ideals is very strong motivation for the following notion, which is studied in this paper:

Definition 1.1. A proper submodule N of M will be called n-almost prime, if for $r \in R$ and $x \in M$ with $rx \in N \setminus (N : M)^{n-1}N$, either $x \in N$ or $r \in (N : M)$. A 2-almost prime submodule will be called an almost prime submodule.

According to definition, each prime submodule is an n-almost prime submodule, for any integer $n > 1$.

In order to obtain our main results, we use some definitions and lemma such as the following:

Lemma 1.2. *[3, Proposition 3.3] and [4, Proposition 3.1] Let M be a multiplications module. If M is non-torsion or finitely generated and I is an ideal of R containing $ann(M)$, then $(IM : M) = I$.*

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*Will be presented in English