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Poster On Benz-planes admitting automorphisms with exactly two fix points pp.: 1–4

On Benz-Planes admitting automorphisms with exactly two fix points

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Abstract

The axiom of Miquel with 8 points plays the same role in Benz-planes as Pappus axiom in projective planes. In this paper we show a Benz plane (i.e. a Möbius, Laguerre or Minkowski plane) that admits automorphisms with two arbitrary fix points satisfies a degenerate form of miquel axiom with 6 points. The converse of this assertion is a part of authors investigations.

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1 Introduction

The geometry of circles is as old as Euclidean geometry. Appollonius problem is the most famous elementary problem in this area. The classical version of Appollonius problem is:

"find all circles tangent to three given circles in the Euclidian plane".

in order to find all solutions we should consider lines as circles with a point at infinity. This idea is well known since Gaus because of his elegant model for Euclidean geometry based on complex plane. Adding an extra point ∞ to all lines in Euclidean plane we find a uniform representation of lines and circles.The objects of this nonlinear geometric structure are points and circles and it is more homogenous than Euclidean plane. This geometry is called classical "*inversive geometry*" or "*Möbius plane*". With stereographic projection this geometry can be consider as the geometry of plane sections (circles) on a sphere in a three dimensional Euclidean space. The geometry of plane sections of a conic in 3-dimensional space [1] gives general structures called Benz planes. Besides Möbius planes, Laguerre planes and Minkowski planes are other types of Benz planes. Walter Benz find in 1970 a uniform analytic definition for them [1]. In Laguerre planes and Minkowski planes we have another type of objects called "generator". We recall an axiomatic definition for all of them.

Let $\mathcal{P}$ be a nonempty set which we call it's elements **"point"**, denoted by capital letters and $\mathfrak{C}$ a nonempty subset of the power set of $\mathcal{P}$ which we call it's elements **" cycle"**, denoted by small letters.

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