Coupled Fixed Points via Measurer of Noncompactness

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Abstract

In this paper, using the technique of measure of nono compactness and Darbo fixed point theorem we prove some theorems on coupled fixed point theorems for a class of functions

Keywords: Keywords: Measure of noncompactness,Banach space, Coupled fixed point
Mathematics Subject Classification [2010]: Subject[2000]: 47H09, 34A12

1 Introduction

Bhaskar and Lakshmikantham [5] introduced the concept of a coupled fixed point for a operator and obtained some coupled fixed point existence theorems for a class of operators. In this paper, using the technique of measure of noncompactness, we prove some the existence theorems of coupled fixed point for a class of operators. Measure of noncompactness have been successfully applied in theories of differential and integral equations(see [7]) . This concept was first introduced by Kuratowski. In some Banach spaces, there are known formulas of measure of noncompactness (see [2]).

Throughout this paper we assume that E is a Banach space. For a subset X of E, the closure and closed convex hull of X in E are denoted by $\bar{X}$, $co(X)$, respectively. Also let $\bar{B}_r$is the closed ball in E centered at zero and with radius r and we write $B(x_0, r)$ to denote the closed ball centered at x_0 with radius r. Moreover, we symbolize by $\mathfrak{M}_E$ the family of nonempty bounded subsets of E and by $\mathfrak{N}_E$ subfamily consisting of all relatively compact subsets of E. In addition to, The norm $\|.\|$ in $E \times E$ is defined by $\|(x, y)\| = \|x\| + \|y\|$ for any $x, y \in E \times E$.

The following definitions will be needed in the sequel.

Definition 1.1. ([3]) A mapping $\mu : \mathfrak{M}_E \longrightarrow [0, \infty)$ is said to be a measure of noncompactness in E if it satisfies the following conditions;

($\mathbf{B}_1$) The family $Ker\mu = \{X \in \mathfrak{M}_E : \mu(X) = 0\}$ is nonempty and Ker$\mu \subseteq \mathfrak{N}_E$.
($\mathbf{B}_2$) If $X \subseteq Y \Rightarrow \mu(X) \leq \mu(Y)$.
(B_3) $\mu(\overline{X}) = \mu(X)$.
(B_4) $\mu(CoX) = \mu(X)$.
(B_5) $\mu(\lambda X + (1-\lambda)Y) \leq \lambda\mu(X) + (1-\lambda)\mu(Y)$ for $\lambda \in [0, 1)$. ($\mathbf{B}_6$) If (X_n) is a sequence of closed sets from $\mathfrak{M}_E$ such that $X_{n+1} \subseteq X_n$, $(n \geq 1)$ and if $\lim_{n\to\infty} \mu(X_n) = 0$, then the intersection set $X_\infty = \bigcap_{n=1}^{\infty} X_n$ is nonempty.

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