Nested splitting conjugate gradient method for solving generalized Sylvester matrix equation

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Abstract

In this paper, a matrix version of a nested splitting conjugate gradient (NSCG) iteration method and its convergence conditions are presented for solving generalized Sylvester matrix equation that coefficient matrices are large and nonsymmetric. This method is inner/ outer iterate, which its inner iterations are CG-like method to approximate each outer iterate, while each outer iteration is induced by a convergent and symmetric positive definite splitting of the coefficient matrices.

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1 Introduction

In this paper, we consider the generalized Sylvester matrix equation

$$\sum_{j=1}^{p} A_j X B_j = C, \tag{1}$$

where $A_j \in \mathbb{R}^{n\times n}, B_j \in \mathbb{R}^{m\times m}, C, X \in \mathbb{R}^{n\times m}$. The generalized Sylvester equation (1) arises in several areas of applications. They play a cardinal role in the control and communication theory and image restoration; for further details see [2].
Note that the linear matrix equation (1) can be reformulated by the following $nm \times nm$ linear system:

$$\mathcal{A}vec(X) = vec(C), \tag{2}$$

where $\mathcal{A} = \sum_{j=1}^{p}(B_j^T \otimes A_j)$. However, it is quite costly and ill-conditioned to solve this linear equation system.
In this paper, we present an iterative method for solving the matrix equation (1) by using the symmetric and skewsymmetric splitting of the matrices A_j and B_j, $j = 1, 2, ..., p$ in a matrix variant of the nested splitting conjugate gradient (NSCG) method, and give sufficient conditions for convergence. In [1], this method proposed for solving the system of

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