On the space of Finslerian metrics

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Abstract

In the present paper, we first prove that the space of Finslerian metrics is an infinite dimensional manifold. Next, we introduce some inner products in the space of Finslerian metrics. Then it is given decomposition for the tangent space of this infinite dimensional manifold by means of Riemannian metric and the Berger-Ebin theorem.

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1 Introduction

Let (M, g) be a connected, compact Finsler manifold. That is, there is a function F on the tangent bundle TM satisfying the following conditions:

- F is a smooth function on the entire slit tangent bundle TM_o.
- F is a positive homogeneous function on the second variable, y.
- The matrix (g_{ij}), $g_{ij}(x,y) = \frac{1}{2}\frac{\partial^2 F^2}{\partial y^i \partial y^j}$ is non-degenerate.

Geodesics of a Finsler structure F are characterized locally by $\frac{d^2x^i}{dt^2} + 2G^i(x, \frac{dx}{dt}) = 0$, where $G^i = \frac{1}{4}g^{ih}(\frac{\partial^2 F^2}{\partial y^h \partial x^j}y^j - \frac{\partial F^2}{\partial x^h})$ are called geodesic spray coefficients. Let $G^i_j = \frac{\partial G^i}{\partial y^j}$ be the coefficients of a nonlinear connection on TM. By means of this nonlinear connection, the tangent space TM_o splits into horizontal and vertical subspaces. TTM_0 spanned by $\{\frac{\delta}{\delta x^i}, \frac{\partial}{\partial y^i}\}$, where $\frac{\delta}{\delta x^i} := \frac{\partial}{\partial x^i} - G^j_i \frac{\partial}{\partial y^j}$ are called Berwald bases and their dual bases are denoted by $\{dx^i, \delta y^i\}$, where $\delta y^i := dy^i + G^i_j dx^j$. Furthermore, this nonlinear connection can be used to define a linear connection called the Berwald connection and its connection 1-forms are defined locally by $\pi^i_j = G^i_{jk}dx^k$ where $G^i_{jk} = \frac{\partial G^i_j}{\partial y^k}$. The connection 1-forms of the Cartan connection are defined by $\tilde{\nabla}\frac{\partial}{\partial x^i} = \omega^j_i \frac{\partial}{\partial x^j}$, where $\omega^i_j = \Gamma^i_{jk}dx^k + C^i_{jk}\delta y^k$ such that

$$\Gamma^i_{jk} = \frac{1}{2}g^{im}(\frac{\partial g_{mj}}{\partial x^k} + \frac{\partial g_{mk}}{\partial x^j} - \frac{\partial g_{kj}}{\partial x^m}) - (C^i_{js}G^s_k + C^i_{ks}G^s_j - C_{kjs}G^{si}),$$

and

$$C^i_{jk} = \frac{1}{2}g^{im}(\frac{\partial g_{mj}}{\partial y^k} + \frac{\partial g_{mk}}{\partial y^j} - \frac{\partial g_{kj}}{\partial y^m}), \tag{1}$$

Hence we have $\tilde{\nabla} = \nabla + \dot{\nabla}$ where, ∇ is the horizontal coeffiecients of the Cartan connection and $\dot{\nabla}$ is the vertical coeffiecients of the Finslerian(Cartan) connection.

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