



Curvature properties and totally geodesic hypersurfaces of some para-hypercomplex Lie groups*

Mansour Aghasi

Isfahan University of Technology

Mehri Nasehi[†]

Isfahan University of Technology

Abstract

In this paper we study some geometrical properties of four-dimensional para-hypercomplex Lie groups. In fact we first explicitly give all totally geodesic hypersurfaces on four types of these homogeneous spaces. Then we investigate Einstein like metrics on these spaces. The existence of four-dimensional para-hypercomplex Lie groups with parallel or cyclic Ricci tensor are also proved.

Keywords: Totally geodesic hypersurfaces, Para-hypercomplex Lie groups, Parallel Ricci tensor

Mathematics Subject Classification [2010]: 53C42, 53C30.

1 Introduction

Hypercomplex and para-hypercomplex structures are interesting structures in mathematics which have many important applications in physics. In [3] Barberis studied four dimensional Lie groups which admit hypercomplex structures and gave a classification for these spaces. Four dimensional real Lie algebras which admit para-hypercomplex structures are classified in [4] by Blazic and Vukmirovic. Then in [7] Salimi Moghaddam considered connected Lie groups corresponding to some of these Lie algebras and gave the exact form of their Levi-Civita connections and sectional curvatures. Also in [1] we have studied harmonicity of invariant vector fields and left-invariant Ricci solitons on these homogeneous spaces. Our aim in this paper is to describe explicitly totally geodesic hypersurfaces on these homogeneous spaces. We also prove the existence of four-dimensional para-hypercomplex Lie groups whose Ricci tensor is parallel or cyclic.

2 Four-dimensional para-hypercomplex Lie groups

Here we report the following classification which is given in [4].

Theorem 2.1. *Up to an isomorphism the only four-dimensional Lie algebras \mathcal{G} admitting an integrable para-hypercomplex structure are either abelian or isomorphic to one of the following Lie algebras*

*Will be presented in English

[†]Speaker