Bernoulli operational matrix for solving optimal control problems

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Abstract

In this paper, we use the Bernoulli operational matrix of derivatives and the collocation points, for solving linear and nonlinear optimal control problems (OCPs). By Bernoulli polynomials bases, the Two-Point Boundary Value Problem (TPBVP), derived from the Pontryagins maximum principle, transforms into the matrix equation.

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1 Introduction

Optimal control problems (OCPs) appear in engineering, science, economics, and many other fields. Since most practical problems are rather too complex to allow analytical solutions, numerical methods are unavoidable for solving these complex practical problems. There are numerous computational methods for solving various practical optimal control problems.

2 Main results

In this paper, we consider following linear optimal control problem (OCP)

$$\begin{aligned} &\dot{x} = Ax(t) + Bu(t),\ x(t_0) = x_0, \\ &J = \frac{1}{2}x(t_f)^T Sx(t_f) + \frac{1}{2}\int_{t_0}^{t_f} (x^T Px + 2x^T Qu + u^T Ru)dt, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathrm{R}^{m\times n}$. The control $u(t)$ is an admissible control if it is piecewise continuous in t for $t \in [t_0, t_f]$. Its values belong to a given closed subset U of $\mathbb{R}^+$. The input $u(t)$ is derived by minimizing the quadratic performance index J, where $S \in \mathbb{R}^{n\times n}$, $P \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times m}$ are positive semi-definite matrices and $R \in \mathbb{R}^{m\times m}$ is positive definite matrix.
we consider Hamiltonian for system (1) as

$$H(x, u, \lambda, t) = \frac{1}{2}(x^T Px + 2x^T Qu + u^T Ru) + \lambda^T (Ax + Bu), \tag{2}$$

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