46th Annual Iranian Mathematics Conference
25-28 August 2015
Yazd University
Poster On solving of the matrix equation $AX = B$ with respect to semi-tensor product pp.: 1–4

On solving of the matrix equation $AX = B$ with respect to semi-tensor product

Badrosadat Afzali Borujeni*
University of Shahrekord

Abstract

This paper studies the solutions of the matrix equation $AX = B$ with respect to semi-tensor product. Firstly, the matrix-vector equation $AX = B$ with semi-tensor product is discussed. Compatible conditions are established for the matrices, and a necessary and sucient condition for the solvability of the matrix-vector equation is proposed and several examples are presented to illustrate the eciency of the results.

Keywords: Matrix equation, matrix-vector equation, semi-tensor product, Kronecker product
Mathematics Subject Classification [2010]: 65-XX, 65FXX, 65F10, 65N22

1 Introduction

In this paper, we study the solutions of the matrix equation $AX = B$ with respect to semi-tensor product, where $A \in M_{m\times n}$, $B \in M_{h\times k}$ are known, and X is to be solved. The semi-tensor product of matrices is proposed by Daizhan Cheng in order to solve linearization problem of nonlinear systems, and a detailed introduction can be found in [1]. Classical matrix theory is good at dealing with bilinear functions, but it can hardly be used for multilinear functions. However, using the semi-tensor product method, a multilinear function can be easily described in a matrix expression. Besides this, the semi-tensor product of matrices is proved to be a powerful tool in many other elds. By semitensor product, a logical system can be converted into an algebraic equation with the same form as a discrete system, and kinds of control problems of logical systems are studied [2]. Moreover semitensor product is well used in game theory [3], nonlinear systems, graph coloring [7] and fuzzy logic systems [4]. During the research, some matrix equations with semi-tensor product are involved.

Definition 1.1. let $A = [a_{ij}] \in M_{m\times n}$ and $B = [b_{ij}] \in M_{p\times q}$. The Kronecker product of A and B is defined as

$$A \otimes B = \begin{pmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \dots & a_{mn}B \end{pmatrix} \in M_{mp\times nq} \tag{1}$$

*Speaker