A classification of cubic one-regular graphs

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Abstract

A graph is *one-regular* if its automorphism group acts regularly on the set of its arcs. In this talk, we classify cubic one-regular graphs of order $2p^2q$.

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1 Introduction

Throughout this paper we consider undirected finite connected graphs without loops or multiple edges. For a graph X we use $V(X)$, $E(X)$ and $\mathrm{Aut}(X)$ to denote its vertex set, edge set and its full automorphism group, respectively. An *s-arc* in a graph is an ordered $(s+1)$-tuple $(v_0, v_1, \cdots, v_{s-1}, v_s)$ of vertices of the graph such that v_{i-1} is adjacent to v_i for $1 \le i \le s$, and $v_{i-1} \neq v_{i+1}$ for $1 \le i \le s-1$. By an n-cycle we shall always mean a cycle with n vertices. Also girth is the length of shortest cycle. For a subgroup $G \le Aut(X)$, a graph X is said to be (G, s)*-arc-transitive* or (G, s)*-regular* if G acts transitively or regularly on the set of s-arcs of X, respectively. In the special case graph is *one-regular* if its automorphism group acts regularly on the set of its arcs.

Proposition 1.1. *Let $p \ge 7$ be a prime and X a cubic symmetric graph of order $2p$. Then X is a one-regular normal Cayley graph on the dihedral group D_{2p}.*

Proposition 1.2. *Let X be a connected cubic symmetric graph and let G be a s-regular subgroup of $Aut(X)$. Then the stabilizer G_v of $v \in V(X)$ in G is isomorphic to $\mathbb{Z}_3, S_3, S_3 \times \mathbb{Z}_2, S_4$ or $S_4 \times \mathbb{Z}_2$ for $s = 1, 2, 3, 4$ or 5, respectively.*

Proposition 1.3. $N_{Aut(X)}(R(G)) = R(G) \rtimes Aut(G, S)$.

Proposition 1.4. *Let G be a finite group and let Q be an abelian Sylow subgroup contained in the center of its normalizer. Then Q has a normal complement K (indeed, K is even a characteristic subgroup of G).*

Proposition 1.5. *The quotient group $N_G(H)/C_G(H)$ is isomorphic to a subgroup of the automorphism group $Aut(H)$ of H.*

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