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New reverse of continuous triangle inequalities type for Bochner integral in Hilbert C*-modules

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Abstract

In this paper some reverses of continuous triangle inequalities for integrable functions with value in a Hilbert C^*-modules are given.

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1 Introduction

Let $f : [a, b] \to K$, $K = \mathbb{C}$ or $\mathbb{R}$ be a Lebesgue integrable function. The following inequality is the continuous version of the triangle inequality

$$\Big|\int_a^b f(x)dx\Big| \leq \int_a^b |f(x)|dx, \tag{1}$$

and plays a fundamental role in mathematical analysis and its applications.
It appears, see [7, p. 492], that the first reverse inequality for (1.1) was obtained by J. Karamata in his book from 1949 [6]:

$$\cos\theta \int_a^b |f(x)|dx \leq \Big|\int_a^b f(x)dx\Big| \tag{2}$$

provided

$$-\theta \leq arg[f(x)] \leq \theta, \qquad x \in [a, b]$$

for given $\theta \in (0, \frac{\pi}{2})$. In [5], S. S. Dragomir has extended the above result for Bochner integrals of vector-valued functions in real or complex Hilbert spaces.
If $(\mathcal{H}; \langle ., .\rangle)$ is a Hilbert space over $K(K = \mathbb{C}, \mathbb{R})$ and $f \in L([a, b]; \mathcal{H})$, this means that $f : [a, b] \to \mathcal{H}$ is strongly measurable on $[a, b]$ and the Lebesgue integral $\int_a^b \|f(t)\|dt$ exists and is finite, and there exist a constant $k \geq 1$ and a vector $e \in H$, $\|e\| = 1$ such that

$$\|f(t)\| \leq kRe\big\langle f(t), e\big\rangle \qquad for \qquad a.e.t \in [a, b] \tag{3}$$

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