Solving a multi-order fractional differential equation using the method of particular solutions

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Abstract

This paper presents a new semi-analytic numerical method for solving multi-order fractional differential equations. The method is based on the use of the particular solutions of the linearized equation. Numerical implementation confirms the validity, efficiency and applicability of the method.

Keywords: Particular solution, Fractional differential equation, Multi-point boundary value problem.
Mathematics Subject Classification [2010]: 34A08, 35M12

1 Introduction

Fractional differential equations have been found to be effective to describe some physical phenomenas. In this paper, the method of particular solutions is applied to solve the multi-order fractional differential equation:

$$D^{\alpha}u(t) = f(t, u(t), D^{\beta_1}u(t), \ldots, D^{\beta_n}u(t)) = 0, \quad u^{(k)}(0) = c_k, \quad k = 0, \ldots, m, \tag{1}$$

where $m < \alpha \leq m+1$, $0 < \beta_1 < \beta_2 < \ldots < \beta_n < \alpha$ and D^{α} denotes Caputo fractional derivative of order α. It should be noted that f can be non linear in general. In Daftardar-Gejji and Jafari [1], it was proved that the Eq.(1) can be represented as a system of fractional differential equations (FDEs)

$$\begin{aligned}
&D^{\alpha_i}u_i(t) = u_{i+1}, \quad i = 1, 2, \ldots, n-1,\\
&D^{\alpha_n}u_i(t) = f(t, u_1, u_2, \ldots, u_n);\\
&u_i^k(0) = c_k^i, \quad 0 \leq k \leq m_i, \quad m_i \leq \alpha_i \leq m_i + 1, \quad 1 \leq i \leq n.
\end{aligned} \tag{2}$$

For more details we refer to [3].

In Section 2, we describe the particular solution method for the solution of multi-point boundary value problems (MPBVPs) and then we present this method to solve multi-order fractional differential equations. A numerical example illustrating the applicability of the method is placed in Section 3.

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