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Unique Path Lifting from Homotopy Point of View and Fibrations

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Abstract

The aim of this paper is to introduce the concepts of path homotopically lifting and its role in the category of fibrations. At first, we have some various notions, closely related to path lifting and unique path lifting; and their relations are supplemented by examples. Then, we study some results in the category of fibration with these notions instead of unique path lifting.

Keywords: Homotopically lifting, Unique path lifting, Fibration
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1 Introduction

A map $p : E \to B$ is called a fibration if it has homotopy lifting property with respect to an arbitrary space X, namely, given maps $\tilde{f} : X \to E$ and $F : X \times I \to B$ such that $F \circ j = p \circ \tilde{f}$ for $j : X \to X \times I$ by $j(x) = (x, 0)$, there is a map $\tilde{F} : X \times I \to E$ such that $\tilde{F} \circ j = \tilde{f}$ and $p \circ \tilde{F} = F$. Also, a map $p : E \to B$ is said to have unique path lifting property (upl) if, given paths w and w' in E such that $p \circ w = p \circ w'$ and $w(0) = w'(0)$, then $w = w'$.

Fibrations with upl, as a generalization of covering spaces are important. It is well known that every fiber (inverse image of a singleton) of a fibration with unique path lifting has no nonconstant path [4, Theorem 2.2.5].

In fact, unique path lifting causes a lot of results about a fibration $p : E \to B$, like injectivity of p_*, uniqueness of lifting of a given map and being homeomorphic of any two fibers [4]. Unique path lifting has an important role in the various topological concepts such as covering theory and new generalizations of covering theory, for example [1, 2, 3]. At first, we consider path lifting in the homotopy category and also will discuss about the uniqueness of this type of path lifting and classical path lifting. In fact, their relations will be introduced by some examples. Then, in the last section we would supplement the relations between these new notions in the presence of fibrations. For example, we call a map $p : E \to B$ has weakly unique path homotopically lifting property (wuphl) if, given paths w and w' in E such that $w(0) = w'(0)$, $w(1) = w'(1)$, $p \circ w \simeq p \circ w'\ rel\ \{0, 1\}$, we have, $w \simeq w'\ rel\{0, 1\}$. We will show that every loop in each fiber of a fibration with wuphl is nullhomotopic, which is a homotopy analogue of the same result when we have unique path lifting. Throughout this paper, a *map* $f : X \to Y$ means a continuous function and $f_* : \pi_1(X, x) \to \pi_1(Y, y)$ will denote the homomorphism induced by f.

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