A New Algorithm to Compute Secondary Invariants

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Abstract

In this paper we present a new method to compute secondary invariants of invariant rings. The main advantage of our approach relies on using SAGBI-Gröbner basis in computation which against the Gröbner basis, keeps the invariant structure of polynomials. For this purpose, we use Molien's formula to compute Hilbert series and find the degree of secondary invariants. When the degrees are known, it is sufficient to compute partial SAGBI-Gröbner bases up to certain degrees to find a set of secondary invariants.

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1 Introduction

Let G be a finite $n \times n$ matrix group, linearly acting on a polynomial ring R with n variables over the field K. The ring of all polynomials in R which are invariant under the action of G is called the invariant ring denoted by R^G, which has also an algebra structure. Thanks to the known Hilbert theorem, R^G is finitely generated as a $K-$algebra and furthermore, there are n algebraically independent homogeneous invariants $P = \{f_1, \ldots, f_n\}$ for which R^G is finitely generated module over sub-algebra $K[f_1, \ldots, f_n]$. The elements of P are called primary invariant, and any minimal system of homogeneous invariants $g_1, \ldots, g_t$ generating R^G as a $K[f_1, \ldots, f_n]-$module is called a system of secondary invariants.

There are some algorithms to compute secondary invariants each of which uses an special kind of Gröbner basis. Most of these algorithms like those stated in [5], use some extra auxiliary variables which increase the volume of computations. Furthermore, Gröbner basis breaks the invariant structure of polynomials. There is a generalization of Gröbner basis for ideals of sub-algebras of polynomial rings, which contains important information about the ideal, and also there are efficient algorithms to compute it [2, 3]. The main idea of this paper is to use SAGBI-Gröbner basis to compute secondary invariants. So, in the sequel we recall necessary concepts and then we state our new algorithm. The following definition states the main computational tool in invariant ring.

Definition 1.1. The Reynolds operator of G is the map $\mathcal{R} : R \to R^G$ mapping each $f \in R$ to $\mathcal{R}(f) = 1/|G|(\sum_{\sigma \in G} f(\sigma.X))$ where X is the column vector of variables.

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