



# Continuous fusion frame and continuous resolution of the identity

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## Abstract

Continuous resolution of the identity (CRI) was introduced, a new family of CRI was constructed, and Moreover, a new operator was then defined for two Bessel continuous fusion sequences and accordingly, a number of reconstruction formulas and a family of CRI were obtained.

**Keywords:** continuous fusion frame, Continuous  $g$ -frame, Continuous resolution of the identity.

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## 1 Introduction

Frames for Hilbert space were formally defined by Duffin and Schaeffer [4] in 1952 for studying some problems in non-harmonic Fourier series. Continuous frames were proposed by G. Kaiser [8] and independently by Ali, Antoine, and Gazeau [2] to a family indexed by some locally compact space endowed by a Radon measure. Abdollahpour and Faroughi [1] introduced the concept of continuous  $g$ -frames as a generalization of discrete  $g$ -frames. Throughout this paper,  $(\Omega, \mu)$  is a measure space,  $H$  and  $K$  are two Hilbert spaces, and  $\{K_\omega\}_{\omega \in \Omega}$  is a sequence of closed Hilbert subspaces of  $K$ . For each  $\omega \in \Omega$ ,  $\mathcal{B}(H, K_\omega)$  is the collection of all bounded linear operators from  $H$  to  $K_\omega$ . We also denote

$$\bigoplus_{\omega \in \Omega} K_\omega = \left\{ \{g_\omega\}_{\omega \in \Omega} : g_\omega \in K_\omega \text{ and } \int_{\Omega} \|g_\omega\|^2 d\mu(\omega) < \infty \right\}$$

**Definition 1.1.** A sequence  $\Lambda := \{\Lambda_\omega \in \mathcal{B}(H, K_\omega) : \omega \in \Omega\}$  is called a continuous  $g$ -frame for  $H$  with respect to  $\{K_\omega\}_{\omega \in \Omega}$ , if

1. for each  $f \in H$ ,  $\{\Lambda_\omega f\}_{\omega \in \Omega}$  is strongly measurable,
2. there are two constants  $0 < A \leq B < \infty$  such that

$$A\|f\|^2 \leq \int_{\Omega} \|\Lambda_\omega f\|^2 d\mu(\omega) \leq B\|f\|^2; \quad (f \in H). \quad (1)$$

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