



Web geometry of Lorentz dynamical system

Rohollah Bakhshandeh-Chamazkoti*

Department of Mathematics, Faculty of Basic Science,
Babol University of Technology, Babol, Iran.

Abstract

The paper is devoted to solve Cartan equivalence problem for a dynamical system that is called Lorenz equations under a web transformation.

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1 Introduction

The method of equivalence of E. Cartan (see [1], [3] and [4]) provides a powerful tool for constructing differential invariants which solve the problem of deciding when two geometric objects are really the same up to some preassigned group of coordinate transformations. In [2] R. B. Gardner gave some examples of solving these problems. For example, he has given the local equivalence problem for $y' = f(x, y)$ under diffeomorphisms of the form $\Phi(x, y) = (\varphi(x), \psi(y))$. We generalize this problem to a system of n first order autonomous ODEs.

We generalize this local equivalence problem to one of the most famous dynamical systems which exhibits chaotic behavior that is the *Lorentz equations*

$$\begin{cases} \dot{x} = -\sigma(x - y) \\ \dot{y} = rx - y - xz \\ \dot{z} = xy - bz \end{cases} \quad (1)$$

where $\sigma, r, b > 0$ and “.” represents derivative with respect to arc length t , under the group of coordinate transformations defined by

$$\Phi(t, x, y, z) = (\xi(t), \varphi_1(x), \varphi_2(y), \varphi_3(z)). \quad (2)$$

that is called the *pseudo-group of web transformations*.

*Speaker