Numerical analysis of fractional differential equation by wavelets

A.H.Refahi Sheikhani
Islamic Azad University,Lahijan Branch

M.Mashoof*
Islamic Azad University,Lahijan Branch

Abstract

In this paper, we introduce methods baised on operational matrix of fractional integration for solving a typical n-term non-homogeneous fractional differential equation (FDE). We use Block-puls, Haar wavletes and Hybrid of Block-pulse functions and shifted Legendre polynomials matrices of fractional integration where a fractional derivative is defined in the Caputo sense. By uses these methods we translate an FDE to an algebric liear equations. Methods has been tested by some numerical examples.

Keywords: FDE, Haar, Blok-pulse, Hybrid function, operational matrices
Mathematics Subject Classification [2010]: 26A33, 97N40

1 Introduction

In recent years, study on application of the FDE in science has attracted increasing attention [5, 6, 7]. For instance, Bagley and Torvik formulated the motion of a rigid plate immersing in a Newtonian fluid[1]. It shoulld be mentioned that the main reasons for the theoretical development are mainly the wide use of polymers in various fields of engineering [8, 9, 11, 12]. An FODE in time domain can be described as the following form,

$$a_n({}_aD_t^{\alpha_n}y(t)) + \cdots + a_1({}_aD_t^{\alpha_1}y(t)) + a_0({}_aD_t^{\alpha_0}y(t)) = u(t), \tag{1}$$

subject to the initial conditions $y^{(i)}(a) = d_i, i = 0, ..., n$, where $a_i \in R, n < \alpha \leq n+1, 0 < \beta_1 < \beta_2 < ... < 1 < \alpha$, and ${}_aD_t^{\alpha_n}y(t)$ denotes the caputo fractional derivative of order α. We begin by introducing some necessary definitions and theorems of the fractional calculus theory. In 1.1 the operational matrices of fractional order integration for some wavelets are obtained. Section 2 is devoted to applying the operational matrices of fractional order integration for solving FODE. Also in 2.2 the proposed methods are applied to an example.

1.1 Definitions and theorems

Definition 1.1. The Riemann-Liouville fractional integral of order α is

$$I^\alpha(f(x)) ={}_a D_t^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-\tau)^{\alpha-1} f(\tau) d\tau \quad ;\alpha > 0 \tag{2}$$

*Speaker