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# A note on the graph of equivalence classes of zero divisors of a ring

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**Abstract**

In this paper we study the graph of equivalence classes of zero divisors of a ring $R$, denoted by $\Gamma_E(R)$. We give some necessary conditions for finiteness of $\Gamma_E(R)$.

**Keywords:** Zero divisor, Annihilator, Associated prime
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## 1 Introduction

The graph of equivalence classes of zero divisors of a ring $R$, denoted by $\Gamma_E(R)$, is defined in [7] and studied in [4]. Let $Z(R)$ denotes the set of zero divisors of a ring $R$ and $Z(R)^* = Z(R)\backslash\{0\}$. Define an equivalence relation $\sim$ on $Z(R)$ as follows[5]: $x \sim y$ if and only if $Ann(x) = Ann(y)$. $\Gamma_E(R)$ is a graph associated to R whose vertices are the classes of elements in $Z(R)^*$, and two distinct classes $[x] \neq [y]$ are joined by an edge if and only if $xy = 0$. Another interpretation of $\Gamma_E(R)$ is as follows: The vertices are the elements of $\{ann(a) : a \in Z(R)^*\}$ and two distinct elements $Ann(x)$ and $Ann(y)$ are adjacent if and only if $xy = 0$.

First we recall some facts and notations related to this paper. Throughout this paper $R$ denotes a commutative ring with unit element. For any ideal $I, Ann(I) = \{r \in R : ri = 0 \forall i \in I\}$ is called an annihilator ideal. We say $R$ satisfies $ACC(Ann)$ if every chain in the set of annihilator ideals has a maximal element. If $R$ is a subring of a Noetherian ring then $R$ satisfies $ACC(Ann)$. A prime ideal $P$ is called an associated prime ideal if $P = Ann(x)$ for some $x \in Z(R)^*$. The set of associated prime ideals of $R$ is denoted by $Ass(R)$. Also a vertex $[x]$ of $\Gamma_E(R)$ is called associated prime if $Ann(x) \in Ass(R)$.

Let $\Gamma$ be a simple graph. The *degree* of $v \in V(\Gamma)$ denoted by $d(v)$. The set of vertices which are adjacent to $v$ is denoted by $N_\Gamma(v)$. A complete subgraph of $\Gamma$ is called a clique. The *clique number* of $\Gamma$, denoted by $\omega(\Gamma)$, is suprimum of size of cliques. A subset $S$ of $V$ is called a dominating set if every vertex in $V\backslash S$ has a neighbor in $S$. The minimum size of the dominating sets is called domination number and is denoted by $\gamma(\Gamma)$.

In [4] and [7] the ring $R$ is Noetherian. In this paper we show that many results are true without Noetherian condition or true with a weaker condition.

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