on prime submodules and hypergraphs

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Abstract

In this paper, for every free R-module F of finite rank, we associate a hypergraph $PH_Q(F)$ called the prime submodules hypergraph of F with respect to Q, where Q is a prime ideal of comutative ring R. We then investigate the interplay between the module-theoretic properties of F and the graph-theoretic properties of $PH_Q(F)$. We also show that $PH_Q(F)$ is the union of Steiner systems and use their properties for counting the number of Q-prime submodules of F when Q is a maximal ideal of R and $[R : Q]$ (number of cosets R in Q) is finite.

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1 Introduction

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Throughout this article, all rings are assumed to be commutative with identity and F denotes a free R-module of finite rank. Let M be an R-module and Q be a prime ideal of R. A proper submodule N of M is called Q-prime if, for $r \in R$, $m \in M$ and $rm \in N$ we have $m \in N$ or $r \in Q = (N : M)$, where $(N : M) = \{r \in R \mid rM \subseteq N\}$. We use the notation $R^{(n)}$ for $\underbrace{R \oplus \cdots \oplus R}_{n-times}$.

A hypergraph is a pair $H = (V, E)$ of disjoint sets where the elements of E are nonempty subsets (of any cardinality) of V. The elements of V are the vertices and the elements of E are the edges of hypergraph. Note that, if the cardinality of each edge is two, then we have a simple graph. For $x \in V$ the degree of x denoted by $d_H(x)$, is the number of edges in E containing x. A hypergraph in which all vertices have the same degree r is said to be regular of degree r or r-regular. A hypergraph is called an intersecting if every pair of edges intersects nontrivially. The hypergraph $H = (V, E)$ is called k-uniform whenever every edge e of H is a $k-$subset of V. A k-uniform hypergraph H is called complete if every k-subset of the vertices is an edge of H. The hypergraph $H' = (V', E')$ is a subhypergraph of the hypergraph $H = (V, E)$, whenever $V' \subset V$ and $E' \subset E$. The union of two hypergraphs H and H' is the hypergraph $H \cup H'$ with $V(H \cup H') = V(H) \cup V(H')$ and $E(H \cup H') = E(H) \cup E(H')$.

Let H be a k-uniform hypergraph. A subset A of $V(H)$ is called a clique of H if every k-subset of A is an edge of H. A path of a hypergraph H is an alternating sequence of

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