Redundancy of Dual Frames

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Abstract

In this manuscript we investigate the ralationship between the redundancies of frames and their duals.

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1 Introduction

Frames for Hilbert spaces have been introduced in 1952 by Duffin and Schaeffer in their fundamental paper [6] and have been studied in the last two decades as a powerful framework for robust and stable representation of signals by introducing redundancy. The customary definition of redundancy was improved by Bodmann, Casazza and Kutyniok in [2] by providing a quantitative measure, which coined upper and lower redundancies. Redundancy is applied in areas such as: filter bank theory [3] , sigma-delta quantization [1], and signal and image processing [4]. Dual frames play an important role in studying frames and their applications, specially in the reconstruction formula. Therefore it is natural to study and consider their redundancy and its relationship with redundancy of original frame. In this paper, we will show that the ratio between redundancies of frames and dual frames is bounded from below and above by some significant numbers. First, we will review the definitions of frames and redundancy function for finite frames .

Definition 1.1. [5] Let $\mathcal{H}$ be a Hilbert space and I be a countable index set. The family $\phi = \{\varphi_i\}_{i\in I}$ in $\mathcal{H}$ is called a frame for $\mathcal{H}$ if there exist constants $0 < A \leq B < \infty$ such that

$$A\|x\|^2 \leq \sum_{i\in I} |\langle x, \varphi_i\rangle|^2 \leq B\|x\|^2 \quad \forall x \in \mathcal{H}.$$

The frame ϕ is called a *tight* frame, if $A = B$ and it is a *Parseval* frame if, $A = B = 1$.

Reader can see [5] for the definitions of synthesis, analysis and frame operators.

For a frame the dual frame (canonical dual frame) defined as follows;

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