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Some fixed point results for the sum of two mappings

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Abstract

In this paper, we obtain some new fixed point theorems for the sum of two weakly sequentialy continuous mappings T_1 and T_2 on an $L-$embedded convex subset C in a Banach space X, in which $T_1 : C \to X$ is nonexpansive and $T_2 : C \to X$ is continuous with $T_2(C)$ being contained in a compact set. As a result, we derive fixed point theorems on weak* compact convex subsets of the continuous dual X^* of an $M-$embedded Banach space X.

Keywords: nonexpansive, fixed point, $L-$embedded, $M-$embedded, weakly sequentialy continuous
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1 Introduction

Let X be a Banach space and C be a subset of X. A mapping $T : C \to X$ is called nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. A point $x \in X$ is called a fixed point of T, if $Tx = x$. A mapping $T : C \to X$ is called compact continuous if T is compact and continuous on C. In [4] O'Regan studied the fixed points of the sum of a nonexpansive mapping with a compact continuous on a weakly compact subset C of X and in [2] and [3] Krasnoselskii combined two well-known fixed point theorems (Schauder's fixed point Theorem and the contraction mapping principle) to gain the fixed points of the sum of two mappings T_1 and T_2 on a closed convex subset C in a Banach space X, in which $T_1 : C \to X$ is a contraction and $T_2 : C \to X$ is continuous with $T_2(C)$ being contained in a compact set . In this paper, among other things we study the fixed point of the sum of two such mapings on an $L-$embedded convex subset of X allowing T_1 to be a nonexpansive mapping instead of a contraction(Theorem 2.2). In [1], Lau and Zhang called a nonempty subset C of a Banach space X, $L-$embedded if there is a subspace X_s of X^{**} such that $X + X_s = X \oplus_1 X_s$ in X^{**} and $\overline{C}^{w^*} \subset C \oplus_1 X_s$. That is, for each $x \in \overline{C}^{w^*}$ there are $c \in C$ and $\xi \in X_s$ such that $x = c + \xi$ and $\|x\| = \|c\| + \|\xi\|$. As remarked in the same paper, (by taking $X_s = 0$) it is readily seen that every $L-$embedded subset C of a Banach space X is weak*$-$closed and hence closed. Also every weakly compact subset of Banach space is $L-$embedded, but not vice-versa, [1].

Next, we use our results to derive fixed point theorems on weak* compact convex subsets of the dual space X^* of an $M-$embedded Banach space X (Theorem 2.4). As in [5], a Banach space X is $M-$embedded if X is an M-ideal in its bidual X^{**}, i.e. $X^{\perp} = \{\varphi \in X^{***} : \varphi(x) = 0 \text{ for all } x \in X\}$ is an l_1-summand in X^{***}.

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