2-absorbing ideal in lattice

Ali Akbar Estaji
University of Hakim Sabzevari

Toktam Haghdadi *
Birjand University of Technology

Abstract

In this paper, we define 2-absorbing, weakly 2-absorbing and n-absorbing ideals in a lattice. We also show that 2-absorbing and weakly 2-absorbing ideals are equvalent in a lattice. It is shown that a non-zero proper ideal I of L is a 2-absorbing ideal if and only if whenever $I_1 \wedge I_2 \wedge I_3 \subseteq I$ then $I_1 \wedge I_2 \subseteq I$ or $I_1 \wedge I_3 \subseteq I$ or $I_2 \wedge I_3 \subseteq I$ for some ideals I_1, I_2, I_3 of L.

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1 Introduction

The concept of 2-absorbing ideals, in a commutative ring, was introduced by A. Badawi, in [1], as a generalization of prime ideals, and some properties of 2-absorbing ideals were studied. The definitions and related threads are taken from [1, 2, 3]. In this paper we introduced the 2-absorbing ideal of a lattice L. A proper ideal I of L is said to be 2-absorbing if $a \wedge b \wedge c \in I$ for $a, b, c \in L$ implies that $a \wedge b \in I$ or $a \wedge c \in I$ or $b \wedge c \in I$.

In this paper we introduce radical of ideal I in a lattice L and we show that $RadI = I$. In Section 2, a 2-absorbing ideal of a lattice L and also a weakly 2-absorbing ideal are defined. Particular, we show that if I be a 2-absorbing ideal, then $|MinI| \leq 2$, where $Min(I)$ denotes the set of minimal prime ideals of I in L.

Then , we introduce the concept n-absorbing ideal in a lattice L. It shown that an n-absorbing ideal is also an m-absorbing ideal for all integers $m \geq n$.

Definition 1.1. Let I be an ideal of a lattice L. The radical of I, denoted $RadI$, is the ideal $\bigcap P$, where the intersection is taken over all prime ideals P which contain I. If the set of prime ideals containing I is empty, then $RadI$ is defined to be L.

Proposition 1.2. *If I is an ideal of a lattice L, then $RadI = I$.*

Definition 1.3. Let I be an ideal of L. A prime ideal P in L is called a minimal prime ideal of I if $I \subseteq P$ and there is no prime ideal P' such that $I \subseteq P' \subset P$.

Proposition 1.4. *If an ideal I of a lattice L is contained in a prime ideal P of a lattice L, then P contains a minimal prime ideal of I.*

Proposition 1.5. [4] *Let I be an ideal of L. Let P be a prime ideal containing I. Then P is a minimal prime ideal belonging to I if and only if for each $x \in P$ there is a $y \notin P$ such that $x \wedge y \in I$.*

*Speaker