



## Risk measure in a financial market

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### Abstract

In this paper we extend the definition of risk measure from  $L^\infty$  to an arbitrary Polish space with special conditions. For this purpose we present a measure preserving transformation between two Polish spaces with special conditions.

**Keywords:** Polish space, Risk measure, Risk management, Transformation

**Mathematics Subject Classification [2010]:** 60Hxx, 60Bxx, 60Gxx

## 1 Introduction

Risk management is a very important concept in financial mathematics and specially in a financial market.

For managing risk in a financial market we need to compute risk measure in a financial market which in [1, 2, 4, 5] is defined on  $\mathbb{L}^\infty$ . In this paper we extend the definition of risk measure from  $\mathbb{L}^\infty$  to an arbitrary uncountable Polish space. For this purpose we construct a measure preserving transformation between two Polish spaces which have special conditions.

## 2 Risk Measure

Risk measure is widely used as instrument to control risk. In fact risk measures assign a real number to a risk in a financial market. As usual in actuarial sciences we assume that  $X$  describes a potential loss, but we allow  $X$  to assume negative values. Let  $(\Omega, \mathcal{F}, P)$  be a probability space and expectation of a random variable  $X$  with respect to  $P$  is denoted by  $E[X]$ .

**Definition 2.1.** [2, 3] Let  $X$  be the set of all functions  $f : \Omega \rightarrow \mathbb{R}$ . A mapping  $\rho : X \rightarrow \mathbb{R}$  is called a risk measure if it has the following conditions.

- Monotonicity: If  $X \leq Y$  then  $\rho(X) \leq \rho(Y)$ ;
- Translation invariance: if  $m \in \mathbb{R}$ , then  $\rho(X + m) = \rho(X) + m$ ;
- Subadditivity:  $\rho(X + Y) \leq \rho(X) + \rho(Y)$ ;
- Positive homogeneity: if  $\lambda > 0$ , then  $\rho(\lambda X) = \lambda \rho(X)$ ;

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